FB-MultiPier

Bridge Software Institute

Bridge Modeling Comparison

FB-MultiPier vs. Manual Calculations®

EXECUTIVE SUMMARY

This report summarizes comparisons between FB-MultiPier (v6.0.0) and manual calculations, in which bearing reactions, as well as both superstructure and substructure displacements are computed using analytical solutions.

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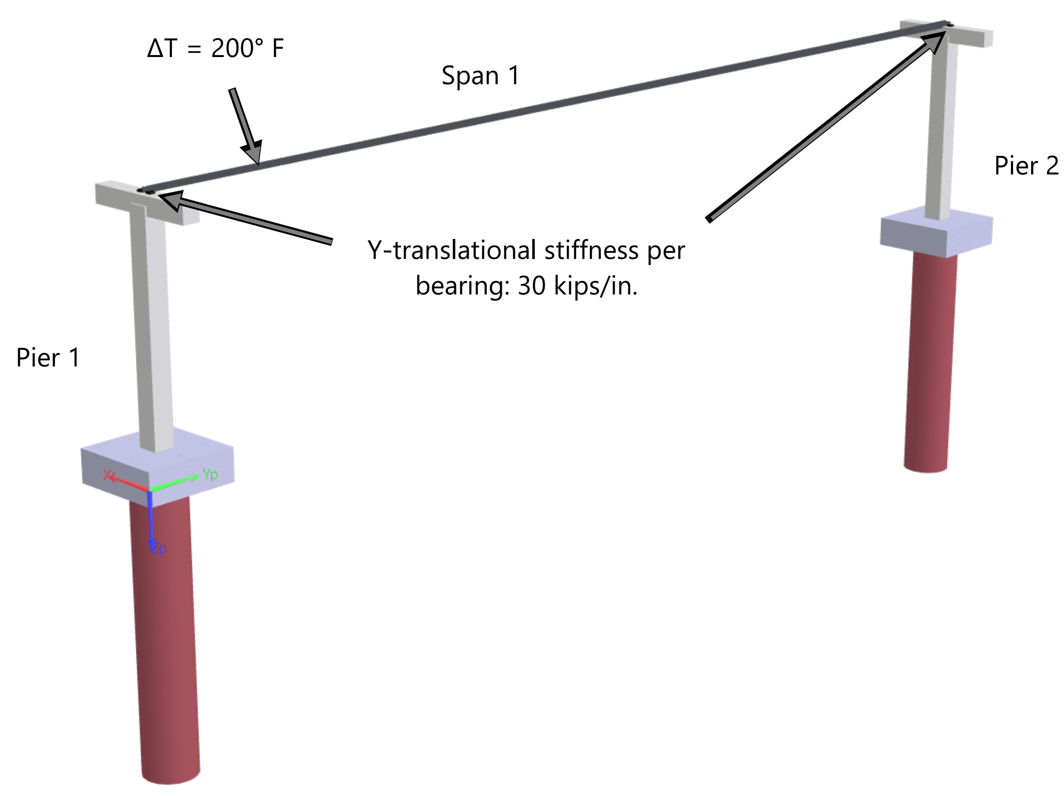
Chapter 1

Simple Span Models with Custom Bearings

In this chapter, simple, two-substructure models are developed with custom (linear) bearings and are subjected to varying loading conditions, resulting in displacements along the global Y axis.

Example 1-1: Uniform Temperature Increase

Problem Description: The bridge piers shown below are not skewed and are simply supported, with two bearings per pier and a span length of 200 ft. For each bearing location, X direction and Z direction translational degrees of freedom are constrained between the pier and superstructure. All rotational degrees of freedom are released. However, for translations in the Y direction, each bearing is assigned a stiffness of 30 kips/in. Pile and pier structural responses in the bridge model are considered linear, and soil is not included in the model (for simplicity). The single bridge span is subjected to a uniform temperature increase of 200° F.



File(s): *Example_1-1.in*

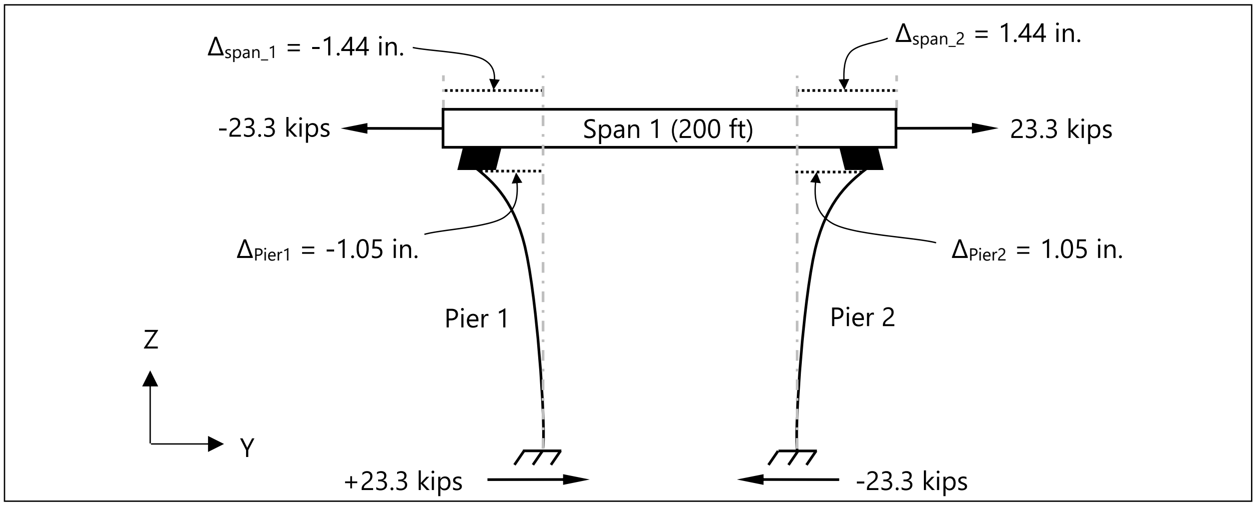


Figure 1.1 – Model Schematic for Example 1.1

Elongation of the bridge span (ΔL) due to the 200° F uniform temperature increase can be calculated as:

$$\Delta L = L_{span} \cdot \alpha \cdot \Delta T$$
$$= 2400 \text{ in.} \cdot 6\text{•E-06 in./in./°F} \cdot 200° \text{ F}$$
$$= 2.88 \text{ in.}$$

where, L_{span} is the span length (in.), α is the coefficient of thermal expansion (in./in./°F), and ΔT is the uniform temperature increase (°F).

Due to symmetry, equal and opposite bridge span displacements of Δ_{span_1} and Δ_{span_2} develop as a result of the imposed span-temperature increase:/

$$\Delta_{span\_1} = -\ \Delta_{span\_2} = -\Delta L/2 = -1.44 \text{ in.}$$

Correspondingly, equal and opposite displacements of Δ_{Pier1} (-1.05 in.) and Δ_{Pier2} (1.05 in.) occur at the center of each pier cap (as obtained from FB-MultiPier). Recall that the stiffness of each bearing pad, for translations in the Y direction, is 30 kips/in. Therefore, the displacement ($\Delta_{bearing1}$) and bearing reaction (R_{B1}) per bearing location at Pier 1, in the Y direction, is:

$$\Delta_{bearing1} = \Delta_{span\_1} - \Delta_{Pier1}$$
$$= -1.44 \text{ in.} + 1.05 \text{ in.}$$
$$= -0.39 \text{ in.}$$

$$R_{B1} = -\ (\Delta_{bearing1}) \cdot (\text{Bearing stiffness})$$
$$= -\ (-0.39 \text{ in.}) \cdot (30 \text{ kips/in.})$$
$$= 11.7 \text{ kips}$$

Similarly, displacement ($\Delta_{bearing2}$) and bearing reaction (R_{B2}) per bearing location at Pier 2 is calculated as 0.39 in. and -11.7 kips respectively. Axial forces that develop in the bridge span frame elements are:

$$F_{axial} = (|\Delta_{bearing1}| + |\Delta_{bearing2}|) \cdot (\text{Bearing stiffness})$$
$$= 0.78 \text{ in.} \cdot 30 \text{ kips/in.}$$
$$= 23.4 \text{ kips}$$

Results from FB-MultiPier Output File (Example_1-1.out):

```
BEARING REACTIONS - Pier #1
Bearing pad reactions are oriented local to the pad rotation

LOC       CASE          FX          FY            FZ          MXX          MYY          MZZ
                      kips        kips          kips       kip-ft       kip-ft       kip-ft
  1          1 -8.2888E-12  1.1655E+01 -1.0000E-01 -2.0898E-09  1.9418E-14  2.7542E-12
  2          1 -8.2470E-12  1.1655E+01 -1.0000E-01 -2.0898E-09 -1.9418E-14 -2.7542E-12
```

Reaction per bearing (FY) = 11.66 kips (matches with R_{B1})

```
BEARING REACTIONS - Pier #2
Bearing pad reactions are oriented local to the pad rotation

LOC       CASE          FX          FY            FZ          MXX          MYY          MZZ
                      kips        kips          kips       kip-ft       kip-ft       kip-ft
  1          1  8.2908E-12 -1.1655E+01 -1.0000E-01  2.0898E-09  9.7094E-15 -1.6227E-12
  2          1  8.2449E-12 -1.1655E+01 -1.0000E-01  2.0898E-09 -9.7094E-15  1.6227E-12
```

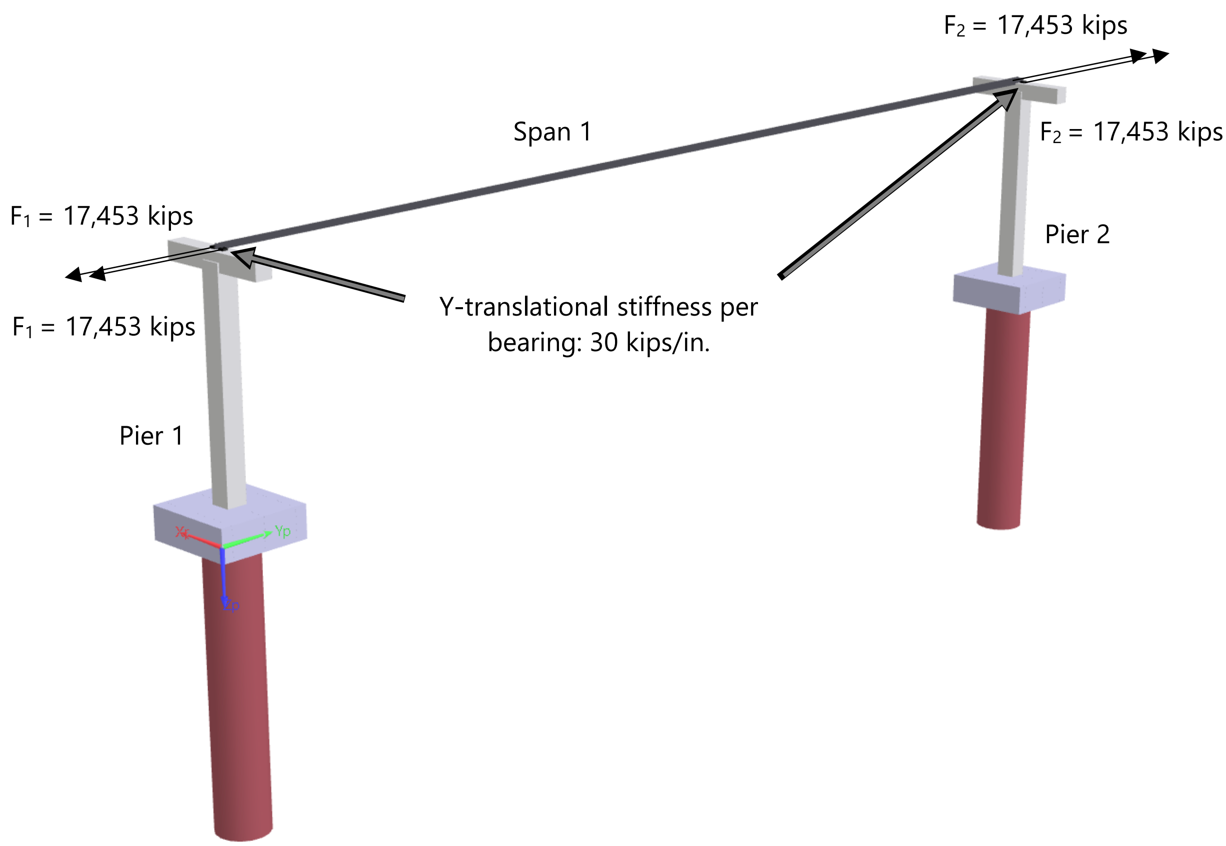
Reaction per bearing (FY) = -11.66 kips (matches with R_{B2})

```
Superstructure Node Displacements
---------------------------------------------------------------
- Span Number  1                                              -
---------------------------------------------------------------
NODE            X            Y            Z          RXX          RYY          RZZ
               in           in           in          rad          rad          rad
   2   -2.2878E-10  -1.4378E+00   1.6931E-05  -7.7740E-08   2.5120E-14  -1.8251E-13
  14    2.0885E-10   1.4378E+00   1.6931E-05   7.7740E-08   2.5102E-14  -1.8242E-13
_______________________________________________________________
Superstructure element forces
---------------------------------------------------------------
- Span Number  1 : Deck                                       -
---------------------------------------------------------------

ELEM NODE  CASE           FAX         F22         F33        M22        M33      TORQUE
 NO.  NO.                kips        kips        kips     kip-ft     kip-ft      kip-ft
   2    2     1     2.331E+01 -1.290E-13 -1.654E-11  9.164E-10 -1.943E-01 -3.487E-12
        3          -2.331E+01  1.290E-13  1.654E-11 -6.410E-10  1.943E-01
```

Bridge span displacement at Pier 1 end = -1.438 in. (matches Δ_{span_1} within 0.1%)
Bridge span displacement at Pier 2 end = 1.438 in. (matches Δ_{span_2} within 0.1%)
Bridge span axial force (FAX) = 23.31 kips (matches F_{axial} within 0.1%)

Example 1-2: Applied Forces

Problem Description: The bridge piers shown below are not skewed and are simply supported, with two bearings per pier and a span length of 200 ft. For each bearing location, X direction and Z direction translational degrees of freedom are constrained between the pier and superstructure. All rotational degrees of freedom are released. However, for translations in the Y direction, each bearing is assigned a stiffness of 30 kips/in. Pile and pier structural responses in the bridge model are considered linear, and soil is not included in the model (for simplicity). Both bearing nodes on the two pier cap beams are subjected to equal and opposite Y-direction forces of 17,453 kips.



File(s): *Example_1-2.in*

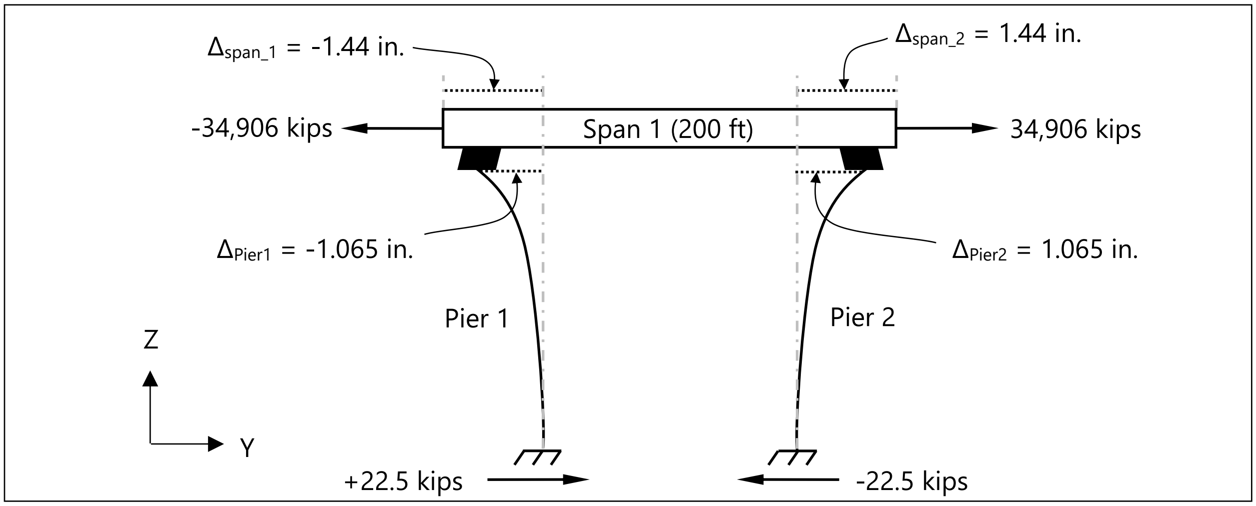


Figure 1.2 – Model Schematic for Example 1.2

Note that the structural configuration of Example 1.2 is identical to that of Example 1.1. Further, recall that for Example 1.1, a uniform temperature increase of 200° F was imposed on the span. For Example 1.2, a comparable set of equal and opposite directly applied forces are calculated and applied to the span ends. The external forces are calculated by relating the elongation of the bridge span (ΔL) due to the 200° F uniform temperature increase to a set of equal and opposite axial forces (F_A):

$\Delta L = L_{span} \cdot \alpha \cdot \Delta T$
$= 2400$ in. · 6•E-06 in./in./°F · 200° F
$= 2.88$ in.

$F_A = (\Delta L) \cdot (A_{span}) \cdot (E_{span}) / (L_{span})$
$= (2.88 \text{ in.}) \cdot (7{,}272 \text{ in.}^2) \cdot (4000 \text{ ksi}) / (2400 \text{ in.})$
$= 34{,}906$ kips

where, L_{span} is the span length (in.), A_{span} is the span cross-sectional area (in.^{2}), E_{span} is the span elastic modulus (ksi), α is the coefficient of thermal expansion (in./in./°F), and, ΔT is the uniform temperature increase (°F).

Given that the Example 1.2 structural configuration contains two bearings atop each pier cap, the as-calculated equal and opposite span-end applied forces are distributed as $F_A/2$ (17,453 kips) at each bearing location. Due to the symmetric configuration of Example 1.2, equal and opposite span displacements of Δ_{span_1} and Δ_{span_2} develop:

$\Delta_{span\_1} = -\ \Delta_{span\_2} = -\Delta L/2 = -1.44$ in.

Correspondingly, equal and opposite pier displacements of Δ_{Pier1} (-1.065 in.) and Δ_{Pier2} (1.065 in.) are developed at the center of each pier cap (as obtained from FB-MultiPier). Recall that the stiffness of each bearing pad, for translations in the Y direction, is 30 kips/in. The displacement ($\Delta_{bearing1}$) and bearing reaction (R_{B1}) per bearing location at Pier 1, in Y direction, can be calculated as:

$\Delta_{bearing1} = \Delta_{span\_1} - \Delta_{Pier1}$
$= -1.44$ in. $+ 1.065$ in.
$= -0.375$ in.

$R_{B1} = -\ (\Delta_{bearing1}) \cdot$ (Bearing stiffness)
$= -\ (-0.375$ in.$) \cdot (30$ kips/in.$)$
$= 11.25$ kips

Similarly, the displacement ($\Delta_{bearing2}$) and bearing reaction (R_{B2}) per bearing location at Pier 2 can be calculated as 0.375 in. and -11.25 kips respectively.

Results from FB-MultiPier Output File (Example_1-2.out):

```
BEARING REACTIONS - Pier #1
Bearing pad reactions are oriented local to the pad rotation

LOC       CASE          FX          FX          FZ          MXX         MYY         MZZ
                        kips        kips        kips        kip-ft      kip-ft      kip-ft
  1          1  8.2423E-12  1.1825E+01  1.5320E-11 -1.7869E-07 -3.4390E-23 -2.2427E-10
  2          1  9.6634E-12  1.1825E+01  1.3389E-11 -1.7869E-07 -5.1005E-25  2.2427E-10
```

Bearing Reaction (FY) = 11.83 kips (matches R_{B1} within 5%)

```
BEARING REACTIONS - Pier #2
Bearing pad reactions are oriented local to the pad rotation

LOC       CASE          FX          FY          FZ          MXX         MYY         MZZ
                        kips        kips        kips        kip-ft      kip-ft      kip-ft
  1          1 -8.1775E-12 -1.1825E+01  2.4447E-12  1.7869E-07  2.4212E-24  2.3460E-10
  2          1 -9.6975E-12 -1.1825E+01 -1.3067E-11  1.7869E-07  7.9415E-24 -2.3460E-10
```

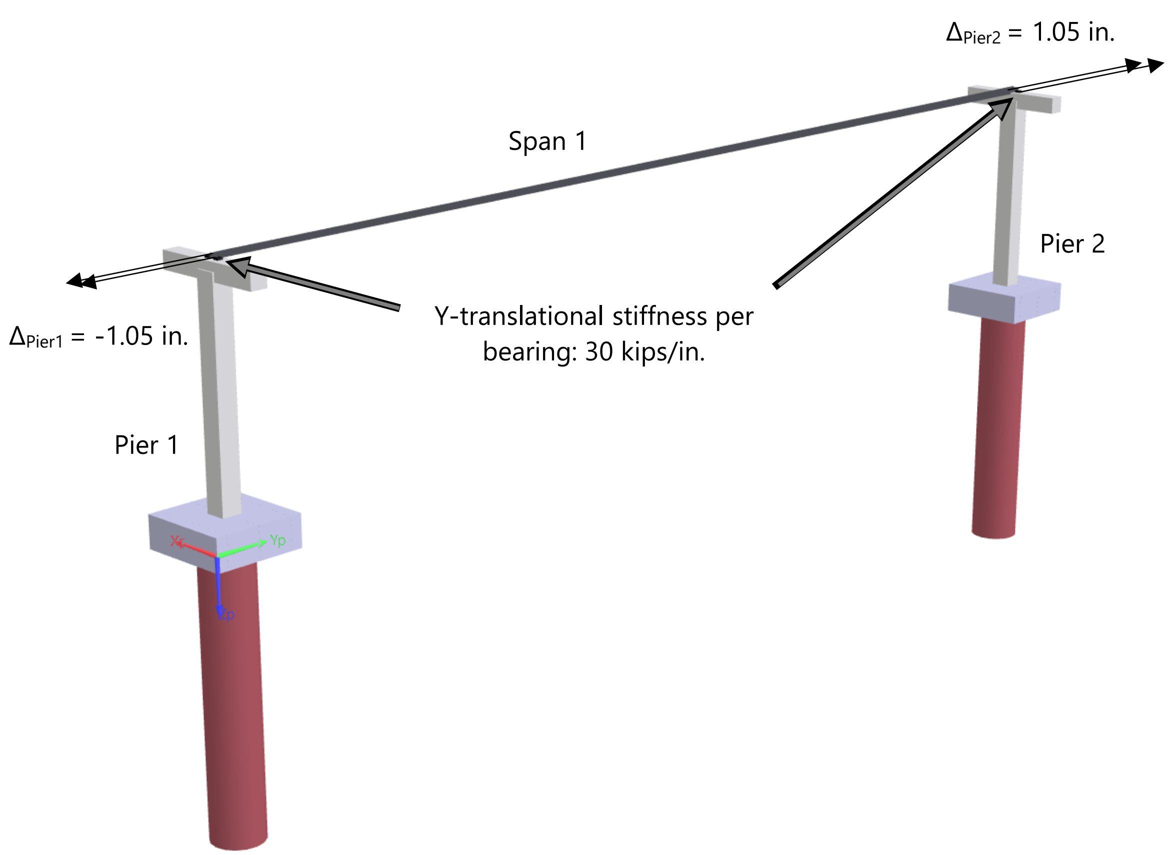
Bearing Reaction (FY) = -11.83 kips (matches R_{B2} within 5%)

```
Superstructure Node Displacements
---------------------------------------------------------------
- Span Number  1                                              -
---------------------------------------------------------------
NODE            X            Y            Z          RXX          RYY          RZZ
               in           in           in          rad          rad          rad
   2   2.0729E-10  -1.4378E+00   1.6583E-13   1.1633E-04   1.7105E-14   1.7812E-13
  14  -2.2089E-10   1.4378E+00  -1.6692E-13  -1.1633E-04   1.7141E-14   1.7892E-13
_______________________________________________________________
Superstructure element forces
---------------------------------------------------------------
- Span Number  1 : Deck                                       -
---------------------------------------------------------------

ELEM NODE  CASE          FAX        F22        F33        M22        M33     TORQUE
 NO.  NO.               kips       kips       kips     kip-ft     kip-ft     kip-ft
   2    2     1   -3.488E+04  2.046E-12  1.787E-11 -8.213E-09  2.907E+02  7.331E-12
        3          3.488E+04 -2.046E-12 -1.787E-11  7.915E-09 -2.907E+02
```

Bridge span displacement at Pier 1 end = -1.438 in. (matches Δ_{span_1} within 0.1%)
Bridge span displacement at Pier 2 end = 1.438 in. (matches Δ_{span_2} within 0.1%)
Bridge span axial force (FAX) = 34,880 kips (matches Total applied force F_A at bearing within 0.1%)

Example 1-3: Prescribed Displacements

Problem Description: The bridge piers shown below are not skewed and are simply supported, with two bearings per pier and a span length of 200 ft. For each bearing location, X direction and Z direction translational degrees of freedom are constrained between the pier and superstructure. All rotational degrees of freedom are released. However, for translations in the Y direction, each bearing is assigned a stiffness of 30 kips/in. Pile and pier structural responses in the bridge model are considered linear, and soil is not included in the model (for simplicity). The center nodes of the two pier cap beams are subjected to equal and opposite prescribed displacements of 1.05 in.



File(s): *Example_1-3.in*

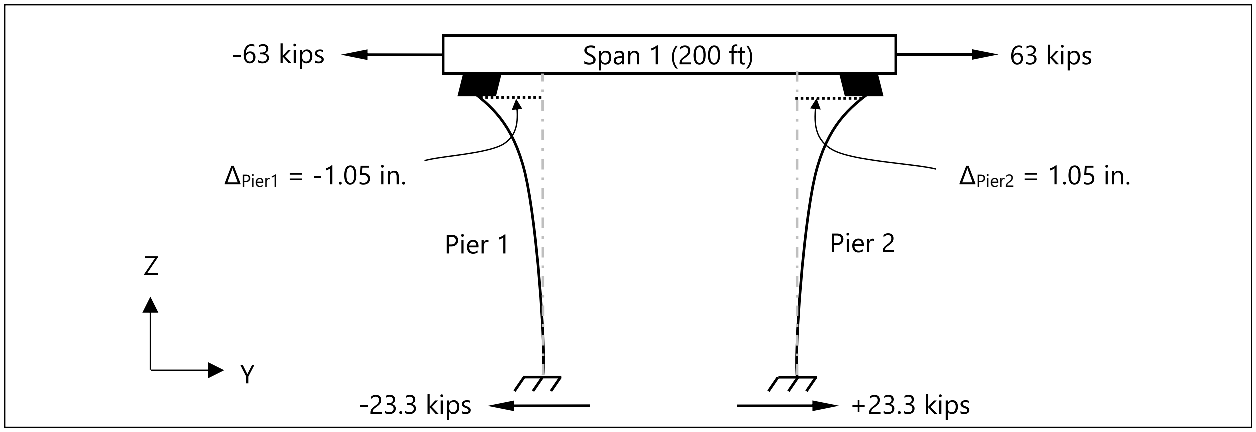


Figure 1.3 – Model Schematic for Example 1.3

Equal and opposite prescribed displacements (Δ_{Pier1} and Δ_{Pier2} of magnitude 1.05 in.) are prescribed at the center of each pier cap, as shown above. The stiffness of each bearing pad, for translations in the Y direction, is 30 kips/in.

The axial force (F_A) in the bridge span is calculated as:

F_A = (Prescribed displacement) • (Bearing stiffness) • (No. of bearings)
= (1.05 in.) • (30 kips/in.) • (2)
= 63.0 kips

Given that the Example 1.3 structural configuration contains two bearings atop each pier cap, the bearing reaction, in the Y direction, per bearing location at Pier 1 is R_{B1} = -63 kips / 2 = -31.5 kips. Similarly, the bearing reaction, in the Y direction, per bearing location at Pier 2 is R_{B2} = 31.5 kips. Recalling Example 1.1, for pier cap displacements of 1.05 in., the magnitude of pile base shear is 23.3 kips, where (as expected) the same magnitude of base shear also develops for the piers of Example 1.3.

Results from FB-MultiPier Output File (Example_1-3.out):

```
BEARING REACTIONS - Pier #1
Bearing pad reactions are oriented local to the pad rotation

LOC        CASE          FX          FX            FZ          MXX          MYY          MZZ
                       kips        kips          kips       kip-ft       kip-ft       kip-ft
  1           1 -1.6854E-10 -3.1192E+01 -2.0393E-12 -3.3562E-09 -8.5349E-23 -5.6097E-11
  2           1  5.6184E-11 -3.1192E+01 -5.1802E-12 -3.3562E-09  1.4948E-22  5.6097E-11
```

Bearing Reaction (FY) = -31.19 kips (matches R_{B1} within 1%)

```
BEARING REACTIONS - Pier #2
Bearing pad reactions are oriented local to the pad rotation

LOC        CASE          FX          FY            FZ          MXX          MYY          MZZ
                       kips        kips          kips       kip-ft       kip-ft       kip-ft
  1           1 -6.3320E-11  3.1192E+01  1.6308E-12  3.3562E-09  2.6597E-23  2.8850E-11
  2           1  3.2093E-11  3.1192E+01  2.8475E-13  3.3562E-09 -6.5713E-23 -2.8850E-11
```

Bearing Reaction (FY) = 31.19 kips (matches R_{B2} within 1%)

```
Superstructure element forces
---------------------------------------------------------------
- Span Number  1 : Deck                                       -
---------------------------------------------------------------

ELEM NODE  CASE            FAX         F22         F33        M22        M33      TORQUE
 NO.  NO.                 kips        kips        kips     kip-ft     kip-ft      kip-ft
   2    2     1     -6.238E+01 -3.331E-15  4.260E-13 -1.134E-10  5.199E-01  2.781E-14
        3            6.238E+01  3.331E-15 -4.260E-13  1.063E-10 -5.199E-01
```

Bridge span axial force (FAX) = 62.38 kips (matches F_A within 1%)

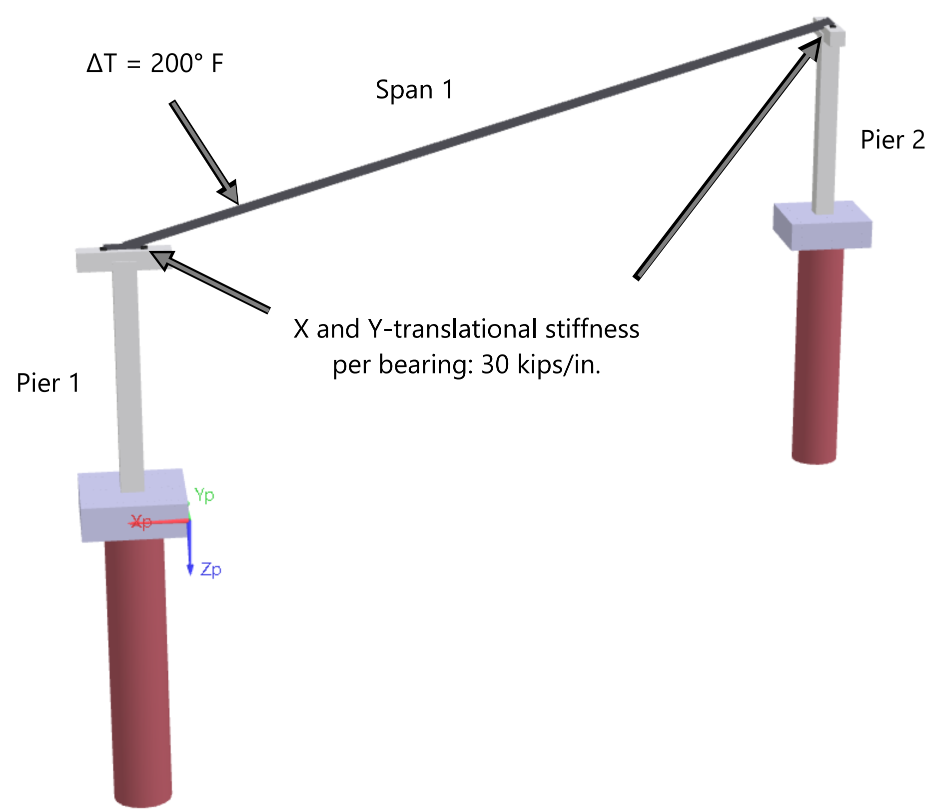
Chapter 2

Simple Span Models with Skewed Custom Bearings and Skewed Piers

In this chapter, simple, two-substructure models are developed with custom (linear) bearings and are subjected to varying loading conditions, resulting in displacements along the global Y axis. All substructures have rotations applied.

Example 2-1: Uniform Temperature Increase

Problem Description: Bridge piers 1 and 2 shown below are skewed at angles of 45° and -45°, respectively, and are simply supported. Two bearings are positioned atop each pier, which are connected via a 200 ft span. All bearings in the model are skewed at an angle of 45°. For each bearing location, Z direction translational degrees of freedom are constrained between the pier and superstructure. All rotational degrees of freedom are released. However, for translations in the local X and Y directions, each bearing is assigned a stiffness of 30 kips/in. Pile and pier structural responses in the bridge model are considered linear, and soil is not included in the model (for simplicity). End nodes of the bridge frame elements are subjected to equal and opposite forces of 34,906 kips in the global Y direction.



File(s): *Example_2-1.in*

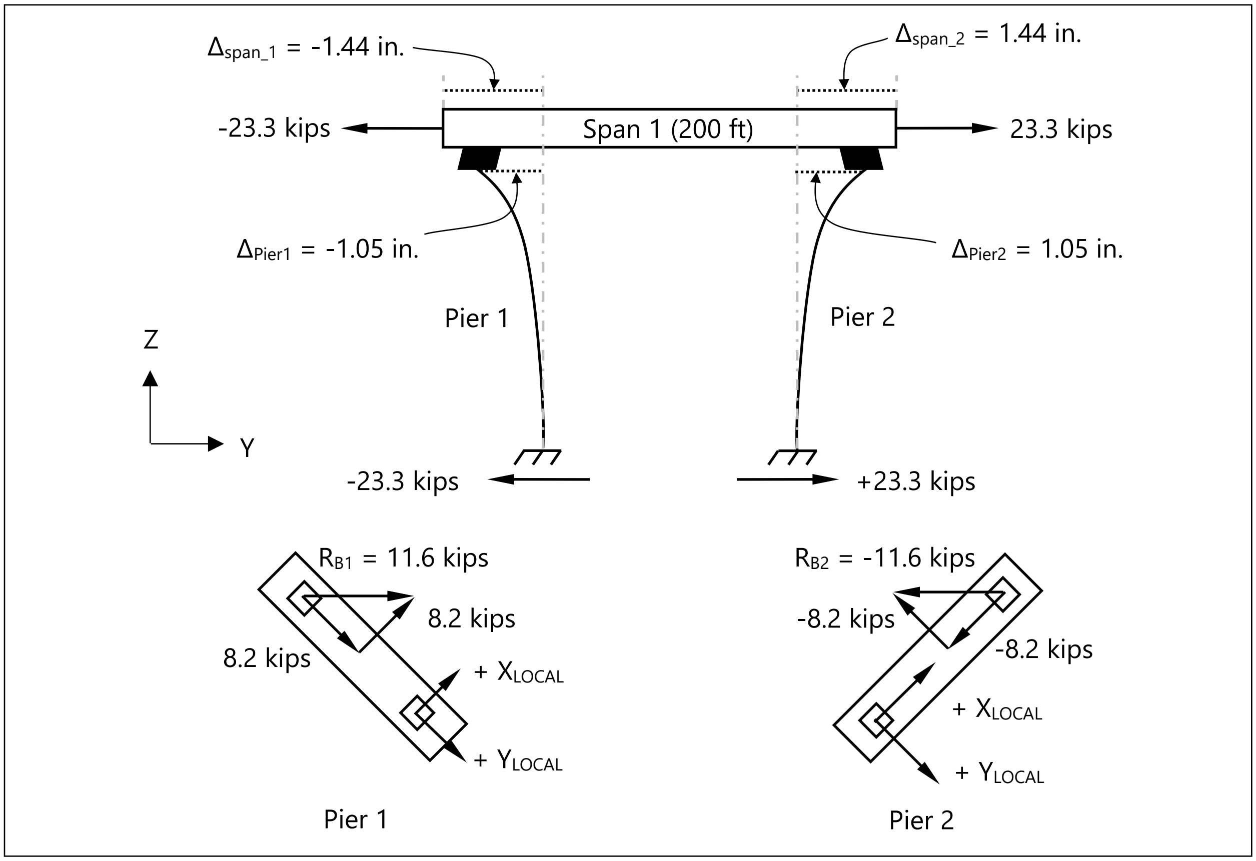


Figure 2.1 – Model Schematic for Example 2.1

Elongation of the bridge span (ΔL) due to the 200° F uniform temperature increase can be calculated as:

$$\Delta L = L_{span} \cdot \alpha \cdot \Delta T$$
$$= 2400 \text{ in.} \cdot 6\text{·E-06 in./in./°F} \cdot 200\text{° F}$$
$$= 2.88 \text{ in.}$$

where, L_{span} is the span length (in.), α is the coefficient of thermal expansion (in./in./°F), and, ΔT is the uniform temperature increase (°F).

Due to symmetry, equal and opposite bridge span displacements of Δ_{span_1} and Δ_{span_2} develop in the global Y direction:

$$\Delta_{span\_1} = -\Delta_{span\_2} = -\Delta L/2 = -1.44 \text{ in.}$$

Correspondingly, equal and opposite displacements of Δ_{Pier1} (-1.05 in.) and Δ_{Pier2} (1.05 in.) develop in the global Y direction at the center of each pier cap (as obtained from FB-MultiPier). Recall that the stiffness of each bearing pad, for translations in the local bearing axes X_{LOCAL} and Y_{LOCAL}, is 30 kips/in. Therefore, the global Y direction displacement ($\Delta_{bearing1}$) per bearing location at Pier 1 is:

$$\Delta_{bearing1} = \Delta_{span\_1} - \Delta_{Pier1} = -1.44 \text{ in.} + 1.05 \text{ in.} = -0.39 \text{ in.}$$

Additionally, the local X and Y direction displacements, $\Delta_{bearing1\_x}$ and $\Delta_{bearing1\_y}$, per bearing location at Pier 1are:

$$\Delta_{bearing1\_x} = \Delta_{bearing1\_y} = \Delta_{bearing1} /\sqrt{2} = -0.39 \text{ in.} / \sqrt{2} = -0.276 \text{ in.}$$

The local X and Y direction bearing reactions, R_{Bx1} and R_{By1}, acting on per bearing location at Pier 1 are:

$R_{Bx1} = -(\Delta_{bearing1\_x}) \cdot (\text{bearing stiffness})$
$= -(-0.276) \cdot (30)$
$= 8.28$ kips

$R_{By1} = -(\Delta_{bearing1\_y}) \cdot (\text{bearing stiffness})$
$= -(-0.276) \cdot (30)$
$= 8.28$ kips

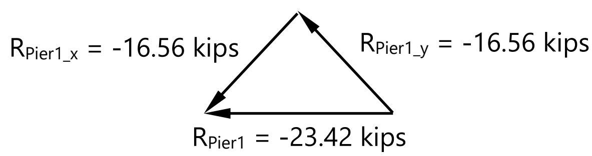
Similarly, the local X and Y direction bearing reactions, R_{Bx2} and R_{By2}, acting on per bearing location at Pier 2 are:

$R_{Bx2} = R_{By2} = -8.28$ kips.

Pile base shears for Pier 1 in the local X and Y directions are:

$R_{Pier1\_x} = -(R_{Bx1}) \cdot (\text{No. of bearings})$
$= -8.28 \text{ kips} \cdot 2$
$= -16.56$ kips

$R_{Pier1\_y} = -(R_{By1}) \cdot (\text{No. of bearings})$
$= -8.28 \text{ kips} \cdot 2$
$= -16.56$ kips



Pile base shears for Pier 2 (in the local X and Y directions) are:

$R_{Pier2\_x} = R_{Pier2\_y} = 16.56$ kips.

Axial forces that develop in the bridge span frame elements are:

$F_{axial} = (R_{Bx1}/\sqrt{2} + R_{By1}/\sqrt{2}) \cdot (\text{No. of bearing})$
$= 8.28/\sqrt{2} + 8.28/\sqrt{2}) \cdot (2)$
$= 23.42$ kips

Results from FB-MultiPier Output File (Example_2-1.out):

```
BEARING REACTIONS - Pier #1
Bearing pad reactions are oriented local to the pad rotation

LOC       CASE          FX          FY           FZ          MXX          MYY          MZZ
                      kips        kips         kips       kip-ft       kip-ft       kip-ft
  1          1  8.2419E+00  8.2432E+00 -1.1275E-02  2.3278E-13 -1.3817E-09  1.4653E-11
  2          1  8.2423E+00  8.2432E+00  1.0806E-02 -1.3700E-13 -1.3817E-09 -1.4369E-11
```

Reaction per bearing (FX & FY) = 8.24 kips (matches R_{Bx1} and R_{By1} within 1%)

```
BEARING REACTIONS - Pier #2
Bearing pad reactions are oriented local to the pad rotation

LOC       CASE          FX          FY           FZ          MXX          MYY          MZZ
                      kips        kips         kips       kip-ft       kip-ft       kip-ft
  1          1 -8.2421E+00 -8.2434E+00  1.1273E-02  2.8883E-09  8.2005E-14 -7.1707E-12
  2          1 -8.2421E+00 -8.2430E+00 -1.0805E-02  2.8883E-09 -9.8806E-14  7.4547E-12
```

Reaction per bearing (FX & FY) = -8.24 kips (matches R_{Bx2} and R_{By2} within 1%)

```
Superstructure Node Displacements
---------------------------------------------------------------
- Span Number  1                                              -
---------------------------------------------------------------
NODE            X            Y            Z          RXX          RYY          RZZ
               in           in           in          rad          rad          rad
   2   -1.3017E-04  -1.4383E+00   1.9515E-05   7.9524E-08  -2.3860E-03  -1.7855E-07
  14    1.0091E-04   1.4383E+00   1.9330E-05   2.3506E-07  -2.3858E-03  -1.7855E-07
_______________________________________________________________
Superstructure element forces
---------------------------------------------------------------
- Span Number  1 : Deck                                       -
---------------------------------------------------------------

ELEM NODE  CASE           FAX        F22        F33       M22        M33      TORQUE
 NO.  NO.                kips       kips       kips    kip-ft     kip-ft      kip-ft
   2    2     1     2.331E+01 -4.686E-04 -1.105E-05  1.104E-03 -2.411E-01  4.684E-02
        3          -2.331E+01  4.686E-04  1.105E-05 -9.203E-04  2.333E-01
```

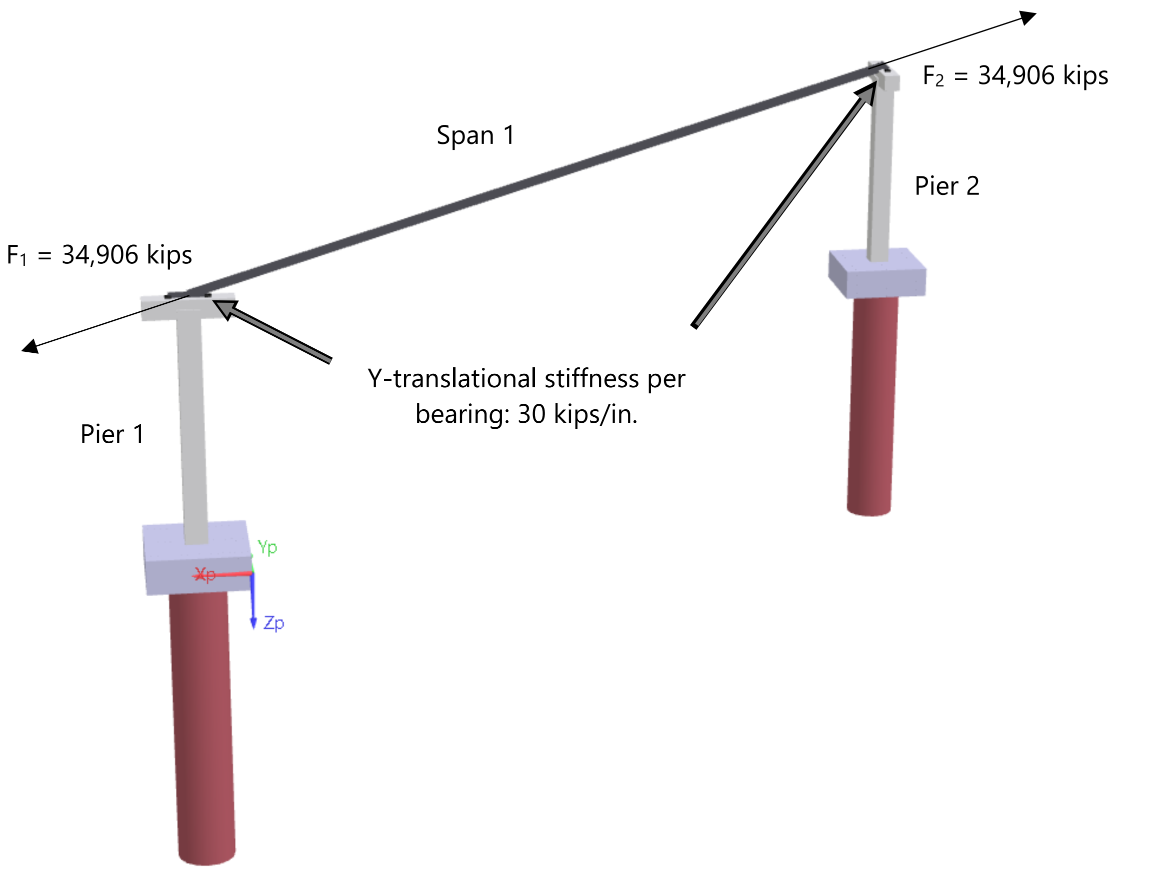
Bridge span displacement at Pier 1 end = -1.44 in. (matches Δ_{span_1} within 0.1%)
Bridge span displacement at Pier 2 end = 1.44 in. (matches Δ_{span_2} within 0.1%)
Bridge span axial force (FAX) = 23.31 kips (matches F_{axial} within 0.5%)

```
Forces Acting on Tip Springs (Pier #1)

Forces are in units of  kips
Moments are in units of kip-ft

PILE NODE LOAD         Zp         Xp         Yp        RZp        RXp        RYp
   1  136    1      0.200      0.002    -23.314     -0.002  -2098.129     -0.036
```

Pile base shear at Pier 1 = -23.31 kips (matches R_{Pier1} within 0.5%)

Example 2-2: Applied Forces

Problem Description: Bridge piers 1 and 2 shown below are skewed at angles of 45° and -45° respectively and are simply supported. Two bearings are positioned atop each pier, which are connected via a 200 ft span. All bearings in the model are skewed at an angle of 45°. For each bearing location, Z direction translational degrees of freedom are constrained between the pier and superstructure. All rotational degrees of freedom are released. However, for translations in the local X and Y directions, each bearing is assigned a stiffness of 30 kips/in. Pile and pier structural responses in the bridge model are considered linear, and soil is not included in the model (for simplicity). End nodes of the bridge frame elements are subjected to equal and opposite forces of 34,906 kips in the global Y direction.



File(s): *Example_2-2.in*

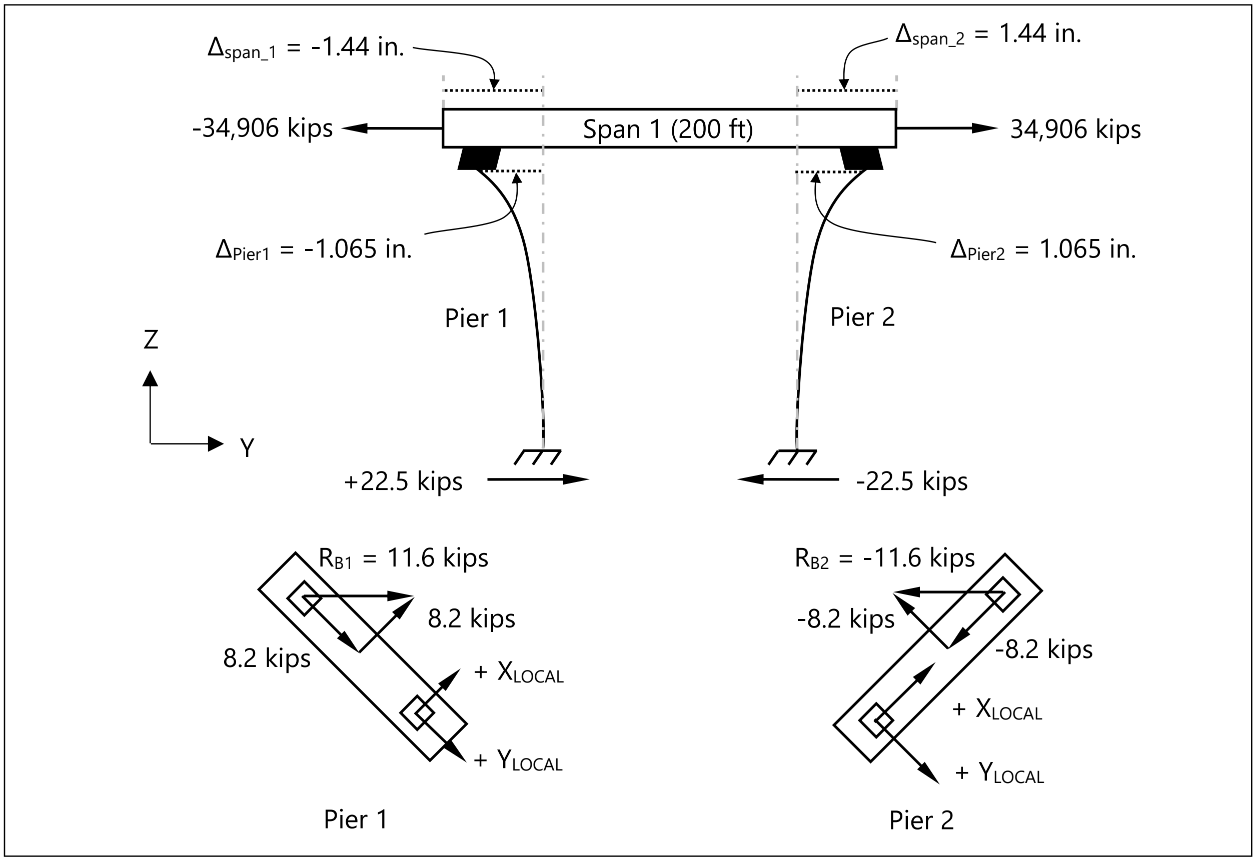


Figure 2.2 – Model Schematic for Example 2.2

Note that the structural configuration of Example 2.2 is identical to that of Example 2.1. Further, recall that for Example 2.1, a uniform temperature increase of 200° F was imposed on the span. For Example 2.2, a comparable set of equal and opposite forces are calculated and applied to the span ends. The external forces are calculated by relating the elongation of the bridge span (ΔL) due to the 200° F uniform temperature increase to a set of equal and opposite axial forces (F_A):

$$\Delta L = L_{span} \cdot \alpha \cdot \Delta T = 2400 \text{ in.} \cdot 6\text{E-06 in./in./°F} \cdot 200\text{° F} = 2.88 \text{ in.}$$

$$F_A = (\Delta L) \cdot (A_{span}) \cdot (E_{span}) / (L_{span}) = (2.88 \text{ in.}) \cdot (7{,}272 \text{ in.}^2) \cdot (4000 \text{ ksi}) / (2400 \text{ in.}) = 34{,}906 \text{ kips}$$

where, L_{span} is the span length (in.), A_{span} is the span cross-sectional area (in.^{2}), E_{span} is the span elastic modulus (ksi), α is the coefficient of thermal expansion (in./in./°F), and ΔT is the uniform temperature increase (°F).

Due to symmetry, equal and opposite bridge span displacements of Δ_{span_1} and Δ_{span_2}, develop in the global Y direction:

$$\Delta_{span\_1} = -\Delta_{span\_2} = -\Delta L/2 = -1.44 \text{ in.}$$

Correspondingly, equal and opposite displacements of Δ_{Pier1} (-1.05 in.) and Δ_{Pier2} (1.05 in.) develop in the global Y direction at the center of each pier cap (as obtained from FB-MultiPier). Therefore, the global Y direction displacement ($\Delta_{bearing1}$) per bearing location at Pier 1 is:

$$\Delta_{bearing1} = \Delta_{span\_1} - \Delta_{Pier1} = -1.44 \text{ in.} + 1.05 \text{ in.} = -0.39 \text{ in.}$$

Additionally, the local X and Y direction displacements, $\Delta_{bearing1\_x}$ and $\Delta_{bearing1\_y}$, per bearing location at Pier 1 are:

$$\Delta_{bearing1\_x} = \Delta_{bearing1\_y} = \Delta_{bearing1} / \sqrt{2} = -0.39 \text{ in.} / \sqrt{2} = -0.276 \text{ in.}$$

Recall that the stiffness of each bearing pad, for translations in the local bearing axis X_{LOCAL} and Y_{LOCAL}, is 30 kips/in. The local X and Y direction bearing reactions, R_{Bx1} and R_{By1}, acting at each of the two bearing locations of Pier 1 are:

$$R_{Bx1} = -(\Delta_{bearing1\_x}) \cdot (\text{bearing stiffness}) = -(-0.276) \cdot (30) = 8.28 \text{ kips}$$

$$R_{By1} = -(\Delta_{bearing1\_y}) \cdot (\text{bearing stiffness}) = -(-0.276) \cdot (30) = 8.28 \text{ kips}$$

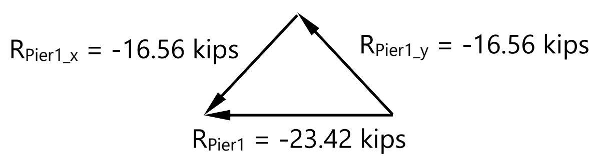
Similarly, the local X and Y direction bearing reactions, R_{Bx2} and R_{By2}, acting at each bearing location of Pier 2 are:

$$R_{Bx2} = R_{By2} = -8.28 \text{ kips.}$$

Pile base shears for Pier 1 in the local X and Y direction are:

$$R_{Pier1\_x} = -(R_{Bx1}) \cdot (\text{No. of bearing}) = -8.28 \text{ kips} \cdot 2 = -16.56 \text{ kips}$$

$$R_{Pier1\_y} = -(R_{By1}) \cdot (\text{No. of bearing}) = -8.28 \text{ kips} \cdot 2 = -16.56 \text{ kips}$$



Similarly, pile base shear for Pier 2 (in the local X and Y direction) are:

$$R_{Pier2\_x} = R_{Pier2\_y} = 16.56 \text{ kips.}$$

Axial forces that develop in the bridge span frame elements are:

$$F_{axial} = F_A = 34,906 \text{ kips}$$

Results from FB-MultiPier Output File (Example_2-2.out):

```
BEARING REACTIONS - Pier #1
Bearing pad reactions are oriented local to the pad rotation

LOC        CASE           FX          FY            FZ          MXX          MYY          MZZ
                        kips        kips          kips       kip-ft       kip-ft       kip-ft
  1           1 -8.2456E+00  8.2453E+00 -1.1277E-02  1.3879E-09  2.5244E-14  1.4656E-11
  2           1 -8.2456E+00  8.2457E+00  1.0809E-02  1.3879E-09  2.2230E-14 -1.4379E-11
```

Reaction per bearing (FX & FY) = 8.25 kips (matches R_{Bx1} and R_{By1} within 1%)

```
BEARING REACTIONS - Pier #2
Bearing pad reactions are oriented local to the pad rotation

LOC        CASE           FX          FY            FZ          MXX          MYY          MZZ
                        kips        kips          kips       kip-ft       kip-ft       kip-ft
  1           1 -8.2455E+00 -8.2458E+00  1.1277E-02  2.8894E-09 -2.1822E-14 -7.1763E-12
  2           1 -8.2455E+00 -8.2454E+00 -1.0809E-02  2.8894E-09 -1.9216E-14  7.4534E-12
```

Reaction per bearing (FX & FY) = -8.25 kips (matches R_{Bx2} and R_{By2} within 1%)

```
Superstructure Node Displacements
---------------------------------------------------------------
- Span Number  1                                              -
---------------------------------------------------------------
NODE            X            Y            Z          RXX          RYY          RZZ
               in           in           in          rad          rad          rad
   2   -1.3935E-04  -1.4389E+00   9.2882E-08  -8.4171E-08  -2.3871E-03  -1.7422E-07
  14    2.7879E-04   1.4389E+00  -9.2855E-08   7.1495E-08  -2.3868E-03  -1.7422E-07
_______________________________________________________________
Superstructure element forces
---------------------------------------------------------------
- Span Number  1 : Deck                                       -
---------------------------------------------------------------

ELEM NODE  CASE           FAX        F22        F33       M22        M33      TORQUE
 NO.  NO.                kips       kips       kips    kip-ft     kip-ft      kip-ft
   2    2     1    -3.488E+04 -4.686E-04 -1.078E-05  1.078E-03 -2.412E-01  4.685E-02
        3           3.488E+04  4.686E-04  1.078E-05 -8.980E-04  2.334E-01
```

Bridge span displacement at Pier1 end = -1.44 in. (matches Δ_{span_1} within 0.1%)

Bridge span displacement at Pier2 end = 1.44 in. (matches Δ_{span_2} within 0.1%)

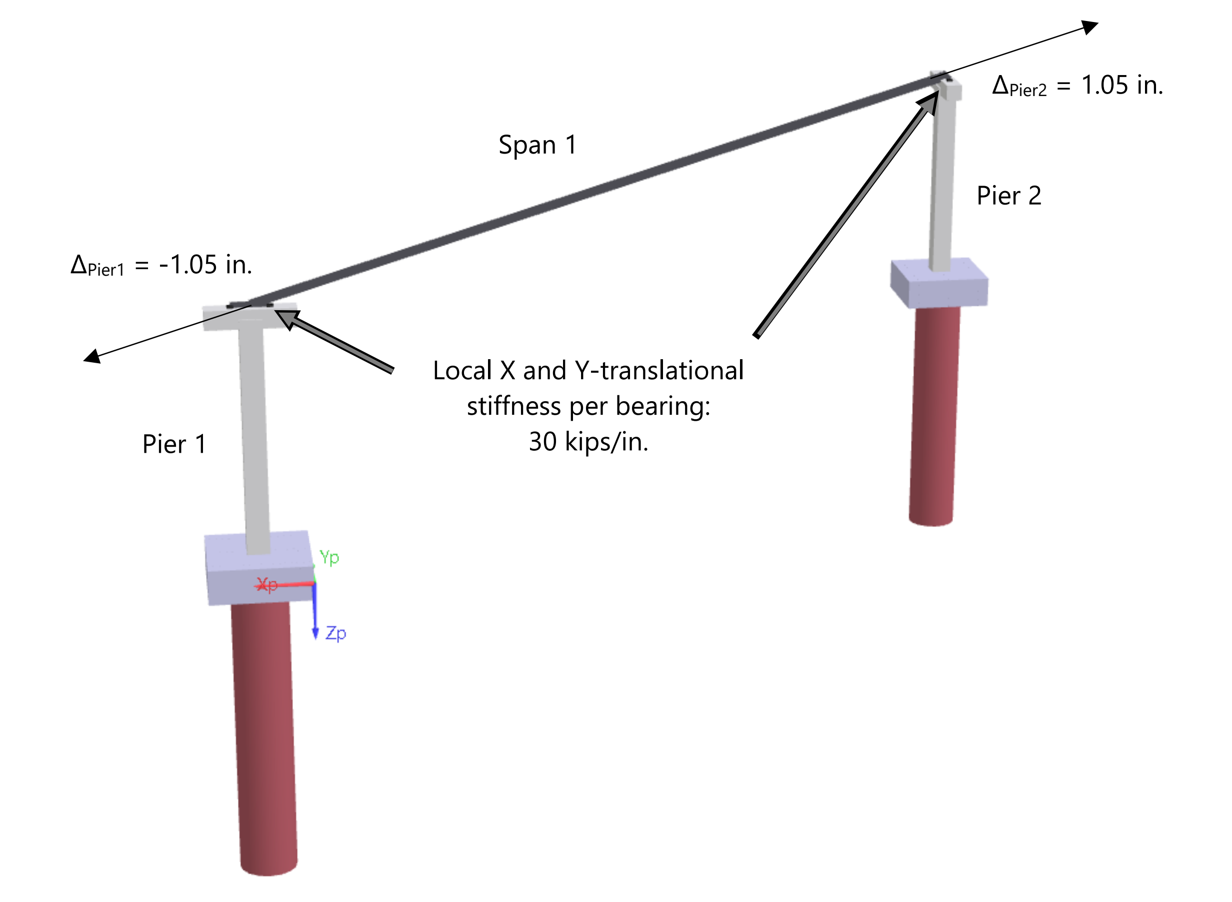
Bridge span axial force (FAX) = 34,880 kips (matches F_{axial} within 0.1%)

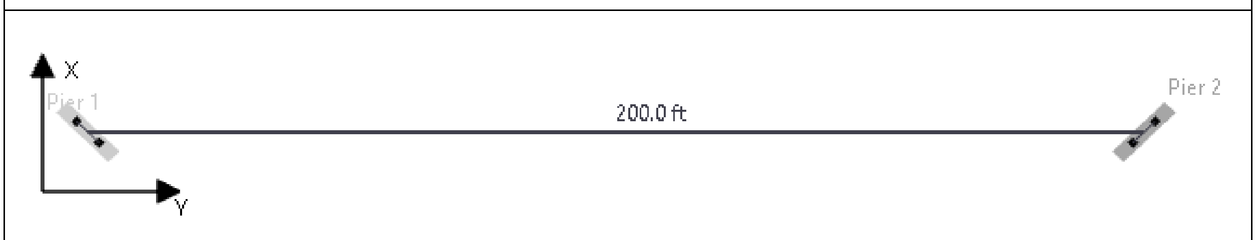
```
Forces Acting on Tip Springs (Pier #1)

Forces are in units of  kips
Moments are in units of kip-ft

PILE NODE LOAD        Zp         Xp         Yp        RZp        RXp        RYp
   1  136    1       0.000      0.000    -23.322     -0.002  -2098.931      0.038
```

Pile base shear at Pier 1 = -23.32 kips (matches R_{Pier1} within 0.5%)

Example 2-3: Prescribed Displacements

Problem Description: Bridge piers 1 and 2 are skewed at angles of 45° and -45° respectively and are simply supported. Two bearings are positioned atop each pier, which are connected via a 200 ft span. All bearings in the model are skewed at an angle of 45°. For each bearing location, Z direction translational degrees of freedom are constrained between the pier and superstructure. All rotational degrees of freedom are released. However, for translations in the local X and Y directions, each bearing is assigned a stiffness of 30 kips/in. Pile and pier structural responses in the bridge model are considered linear, and soil is not included in the model (for simplicity). The center nodes of the two pier cap beams are subjected, in the global Y direction, to equal and opposite prescribed displacements of 1.05 in.





File(s): *Example_2-3.in*

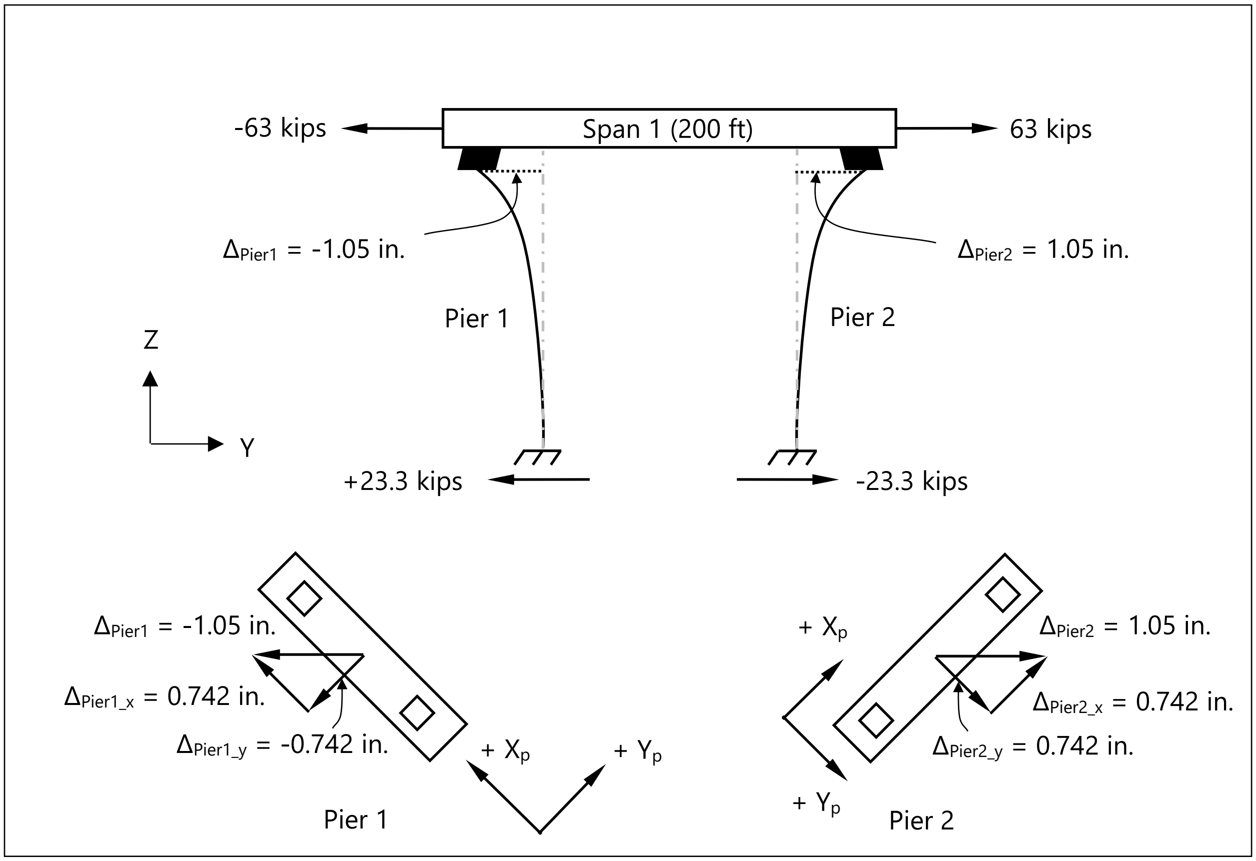


Figure 2.3 – Model Schematic for Example 2.3

Prescribed displacements (Δ_{Pier1_x} and Δ_{Pier1_y} of magnitude 0.742 in.) are imposed in the local X and Y directions at the center of pier cap at Pier 1. Similarly, prescribed displacements Δ_{Pier2_x} and Δ_{Pier2_y} are imposed at Pier 2.

The resultant prescribed displacements, in the global Y direction, at the center of each pier cap are:

$$\Delta_{Pier1} = -\Delta_{Pier2} = -1.05 \text{ in.}$$

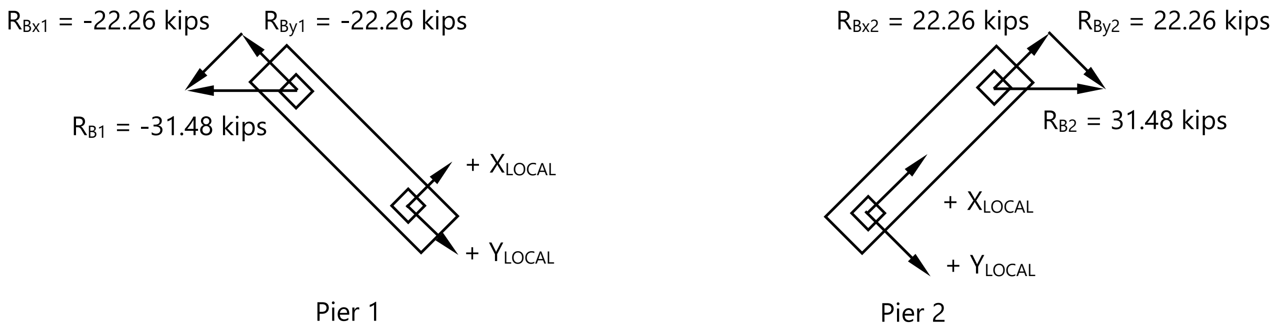
Recall that the stiffness of each bearing pad (for translations in local X and Y directions) is 30 kips/in. Therefore, the axial force (F_A) in the bridge span is calculated as:

$$\begin{aligned} F_A &= (\text{Bearing reaction in global Y direction}) \cdot (\text{No. of bearings}) \\ &= [(0.742 \text{ in.} \cdot 30 \text{ kips/in.})/\sqrt{2} + (0.742 \text{ in.} \cdot 30 \text{ kips/in.})/\sqrt{2})] \cdot (2) \\ &= (1.05 \text{ in.}) \cdot (30 \text{ kips/in.}) \cdot (2) \\ &= 63.0 \text{ kips} \end{aligned}$$

The local X and Y direction bearing reactions, R_{Bx1} and R_{By1}, acting on per bearing location at Pier 1 are:

$$\begin{aligned} R_{Bx1} &= -(\text{Prescribed Disp.}) \cdot (\text{bearing stiffness}) \\ &= -(0.742) \cdot (30) \\ &= -22.26 \text{ kips} \end{aligned}$$

$$\begin{aligned} R_{By1} &= -(\text{Prescribed Disp.}) \cdot (\text{bearing stiffness}) \\ &= -(0.742) \cdot (30) \\ &= -22.26 \text{ kips} \end{aligned}$$



Similarly, the local X and Y direction bearing reactions, R_{Bx2} and R_{By2}, acting on per bearing location at Pier 2 are:

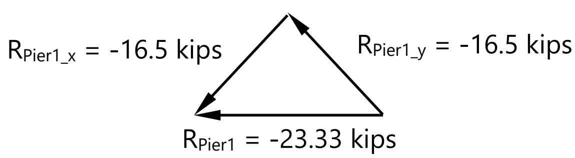
$R_{Bx2} = R_{By2} = 22.26$ kips.

Recalling Example 2-1, for a pier cap displacement of 1.05 in. at Pier 1, the magnitude of pile base shear (R_{Pier1}), in the global Y direction, is -23.3 kips, where (as expected) the same magnitude of base shear also develops for Pier 1 of Example 2-3.

Pile base shears for Pier 1, in the local X and Y direction, are:

$R_{Pier1\_x} = (R_{Pier}) / \sqrt{2}$
$= -23.33 \text{ kips} / \sqrt{2}$
$= -16.5$ kips

$R_{Pier1\_y} = (R_{Pier}) / \sqrt{2}$
$= -23.33 \text{ kips} / \sqrt{2}$
$= -16.5$ kips



Similarly, pile base shear for Pier 2, in the local X and Y direction, is:

$R_{Pier2\_x} = R_{Pier2\_y} = 16.5$ kips.

Axial forces that develop in the bridge span frame elements are:

$F_{axial} = F_A = 34{,}906$ kips

Results from FB-MultiPier Output File (Example_2-3.out):

```
BEARING REACTIONS - Pier #1
Bearing pad reactions are oriented local to the pad rotation

LOC        CASE          FX          FY            FZ         MXX          MYY          MZZ
                       kips        kips          kips      kip-ft       kip-ft       kip-ft
  1           1 -2.2195E+01 -2.2199E+01  3.0362E-02 -6.7911E-14  1.5253E-08 -3.9300E-11
  2           1 -2.2196E+01 -2.2199E+01 -2.9100E-02 -5.9774E-14  1.5253E-08  3.8857E-11
```

Reaction per bearing (FX and FY) = -22.20 kips (matches R_{Bx1} and R_{By1} within 1%)

```
BEARING REACTIONS - Pier #2
Bearing pad reactions are oriented local to the pad rotation

LOC        CASE          FX          FY            FZ         MXX          MYY          MZZ
                       kips        kips          kips      kip-ft       kip-ft       kip-ft
  1           1  2.2196E+01  2.2199E+01 -3.0358E-02  3.7541E-09  5.8690E-14  1.9471E-11
  2           1  2.2196E+01  2.2198E+01  2.9096E-02  3.7541E-09  5.1660E-14 -1.9915E-11
```

Reaction per bearing (FX and FY) = 22.20 kips (matches R_{Bx2} and R_{By2} within 1%)

```
Superstructure element forces
---------------------------------------------------------------
- Span Number  1 : Deck                                       -
---------------------------------------------------------------

ELEM NODE  CASE            FAX         F22         F33        M22        M33      TORQUE
 NO.  NO.                 kips        kips        kips     kip-ft     kip-ft      kip-ft
   2    2     1    -6.278E+01  1.262E-03  1.725E-05 -1.724E-03  6.493E-01 -1.261E-01
        3           6.278E+01 -1.262E-03 -1.725E-05  1.436E-03 -6.283E-01
```

Bridge span axial force (FAX) = 62.78 kips (matches F_{axial} within 0.5%)

```
Forces Acting on Tip Springs (Pier #1)

Forces are in units of  kips
Moments are in units of kip-ft

PILE NODE LOAD         Zp         Xp          Yp         RZp         RXp         RYp
   1  136    1     -0.001     -0.004     -23.322       0.002  -2099.151       0.218
```

Pile base shear at Pier 1 = -23.32 kips (matches R_{Pier1} within 0.5%)

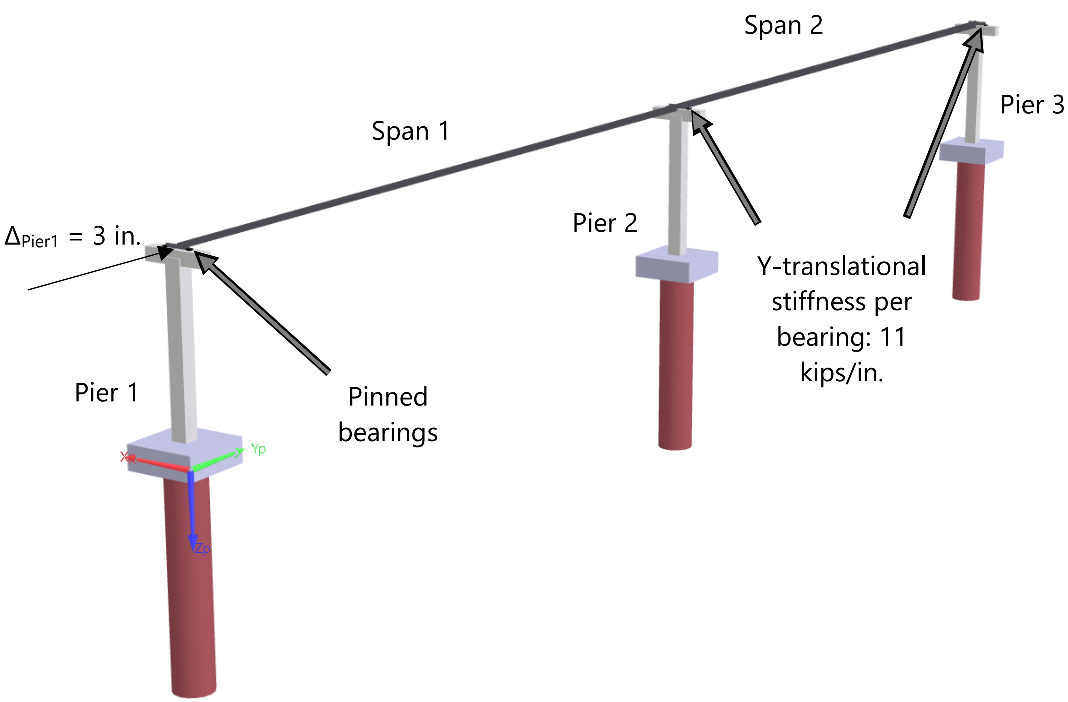
Chapter 3

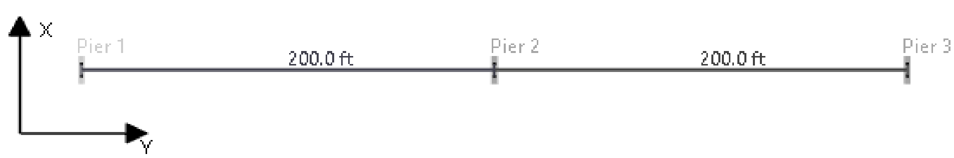
Two-Span Models

In this chapter, three-substructure (two-span) models are developed with both pinned and custom (linear) bearings and are subjected to varying loading conditions, resulting in displacements along the global Y axis.

Example 3-1: Prescribed Displacements – One Row of Bearings

Problem Description: The bridge piers shown below are not skewed and are simply supported, with two bearings per pier and span lengths of 200 ft. At Pier 1, the bearings are pinned. For Pier 2 and Pier 3, the bearings are constrained (between the pier and superstructure) for the global X direction and Z direction translational degrees of freedom. However, for translations in the global Y direction, each bearing of Pier 2 and Pier 3 is assigned a stiffness of 11 kips/in. All rotational degrees of freedom are released. Pile and pier structural responses in the bridge model are considered linear, and soil is not included in the model (for simplicity). The center node on pier cap at Pier 1 is subjected to, in the global Y direction, a prescribed displacement of 3 in.





File(s): *Example_3-1.in*

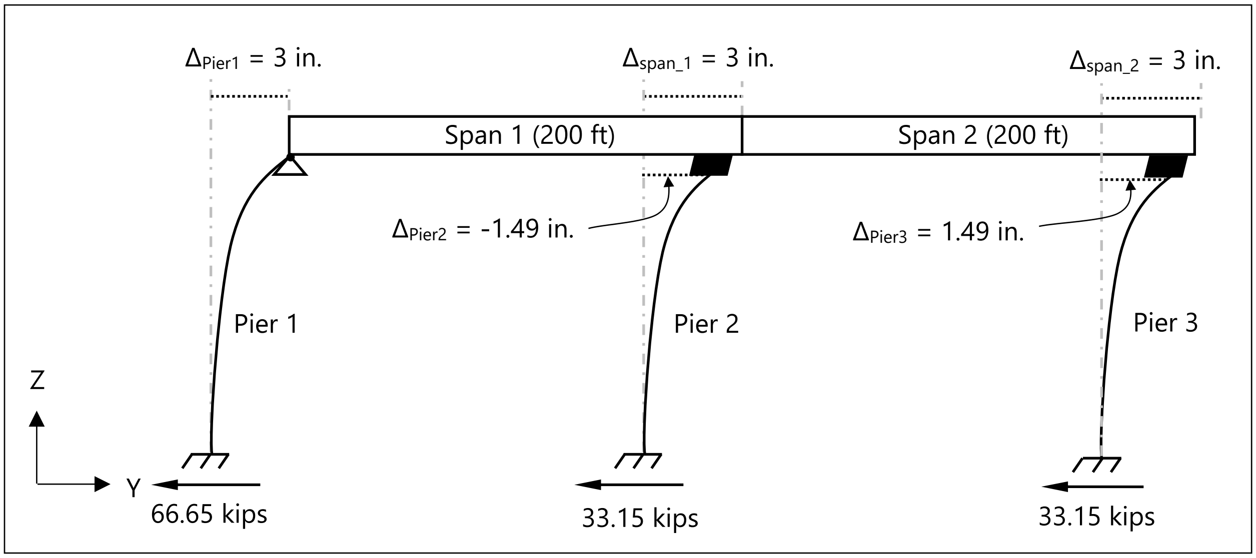


Figure 3.1 – Model Schematic for Example 3.1

A prescribed displacement (Δ_{Pier1} = 3 in.) is imposed in the global Y direction at the center of the Pier 1 pier cap. The prescribed displacement causes a pile base shear of R_{Pier1} = 66.65 kips (as obtained from FB-MultiPier). The stiffness of Pier 1 in the global Y direction is calculated as:

$K_{Pier1} = R_{Pier1} / \Delta_{Pier1}$ = 66.65 kips / 3 in. = 22.22 kips/in.

The structural configuration of Pier 2 is identical to that of Pier 1. Additionally, in the global Y direction, each of the two bearings atop Pier 2 is assigned a stiffness of 11 kips/in. The total stiffness of Pier 2 in the global Y direction is:

$1/ K_{Pier2} = 1/ K_{Pier1} + 1/ (2 \cdot 11$ kips/in.) = 1/ (22.22 kip/in.) + 1/ (22 kips/in.)
K_{Pier2} = 11.05 kips/in.

Similarly, the stiffness of Pier 3 in the global Y direction is:

K_{Pier3} = 11.05 kips/in.

Due to the prescribed displacement, bridge span 1 (Δ_{span1}) and span 2 (Δ_{span2}), displace 3 in. (in the global Y direction):

$\Delta_{span1} = \Delta_{span2}$ = 3 in.

The corresponding pile base shear for Pier 2 (in the global Y direction) is:

R_{Pier2} = (Stiffness of Pier 2, K_{Pier2}) $\cdot$ (Prescribed displacement at span end)
= (11.05 kips/in.) $\cdot$ (3 in.)
= 33.15 kips

Similarly, pile base shear for Pier 3 (in the global Y direction) is:

R_{Pier2} = 33.15 kips

Displacements that develop in the global Y direction at the pier caps of Pier 2 and Pier 3 (as obtained from FB-MultiPier) are:

$\Delta_{Pier2} = \Delta_{Pier3}$ = 1.49 in.

Therefore, the global Y direction displacement ($\Delta_{bearing2}$) per bearing location at Pier 2 is:

$\Delta_{bearing2} = \Delta_{span2} - \Delta_{Pier2}$ = 3 in. – 1.49 in. = 1.51 in.

Recall that the stiffness of each bearing pad, for translation in the global Y direction, is 11 kips/in. The corresponding bearing reaction (R_{B2}) acting per bearing location at Pier 2 is:

R_{B2} = - ($\Delta_{bearing2}$) • (bearing stiffness)
= - (1.51 in.) • (11 kips/in.)
= -16.61 kips

Similarly, the global Y direction bearing reaction (R_{B3}) acting per bearing location at Pier 3 is:

R_{B3} = -16.61 kips

Results from FB-MultiPier Output File (Example_3-1.out):

```
BEARING REACTIONS - Pier #2
Bearing pad reactions are oriented local to the pad rotation

LOC       CASE          FX         FY           FZ          MXX          MYY          MZZ
                      kips       kips         kips       kip-ft       kip-ft       kip-ft
  1          1  1.2779E-09 -1.6543E+01  3.0897E-03  6.0996E-09 -2.2587E-14 -2.9144E-11
  2          1  3.6450E-09 -1.6543E+01  3.0897E-03  6.0996E-09  2.2587E-14  2.9144E-11
```

Reaction per bearing (FY) = -16.54 kips (matches R_{B2} within 1%)

```
BEARING REACTIONS - Pier #3
Bearing pad reactions are oriented local to the pad rotation

LOC       CASE          FX         FY           FZ          MXX          MYY          MZZ
                      kips       kips         kips       kip-ft       kip-ft       kip-ft
  1          1 -2.1092E-09 -1.6528E+01 -1.5449E-03  2.9635E-09  9.9438E-15 -1.4675E-11
  2          1  1.3973E-10 -1.6528E+01 -1.5449E-03  2.9635E-09 -9.9438E-15  1.4675E-11
```

Reaction per bearing (FY) = -16.53 kips (matches R_{B3} within 1%)

```
Forces Acting on Tip Springs (Pier #2)

Forces are in units of  kips
Moments are in units of kip-ft

PILE NODE LOAD         Zp         Xp         Yp         RZp         RXp         RYp
   1  136    1     -0.006      0.000     33.086       0.000    2977.748      -0.000
```

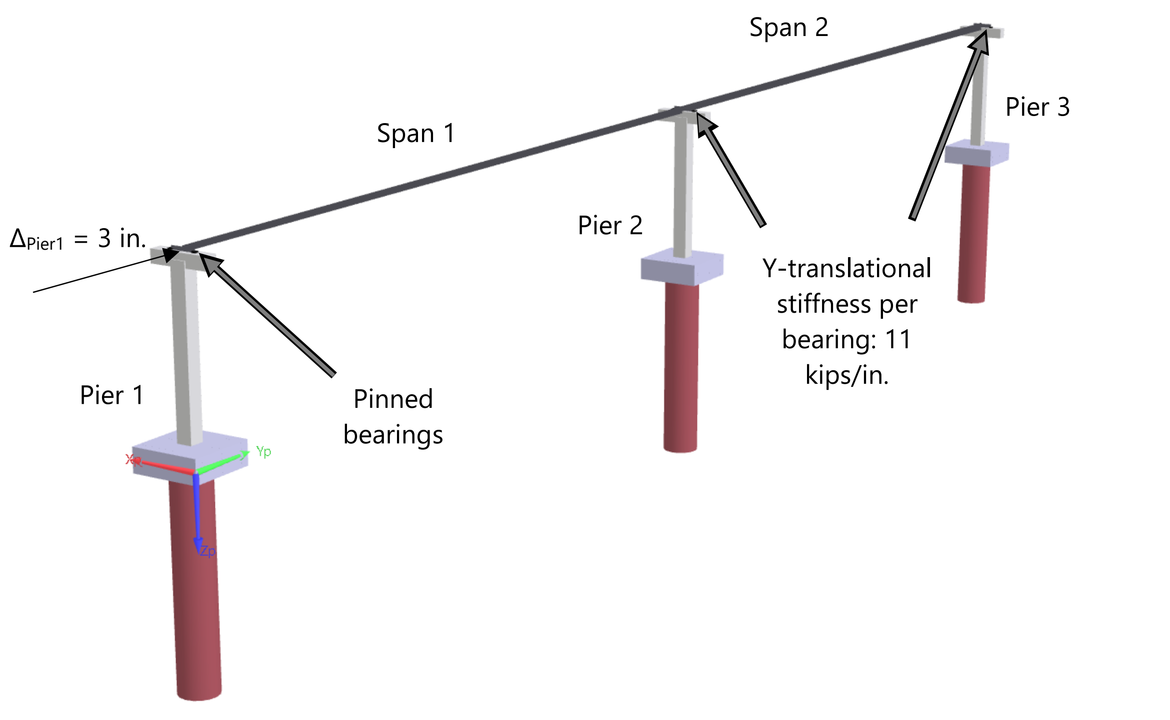
Pile base shear at Pier 2 = 33.09 kips (matches R_{Pier2} within 0.5%)

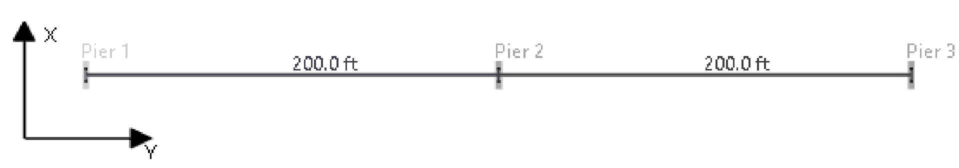
```
Forces Acting on Tip Springs (Pier #3)

Forces are in units of  kips
Moments are in units of kip-ft

PILE NODE LOAD         Zp         Xp         Yp         RZp         RXp         RYp
   1  136    1      0.003      0.000     33.056       0.000    2975.036      -0.000
```

Pile base shear at Pier 3 = 33.06 kips (matches R_{Pier3} within 0.5%)

Example 3-2: Prescribed Displacements – Continuous Spans

Problem Description: The bridge piers shown below are not skewed and are simply supported, with continuous span lengths of 200 ft. Two bearings located atop Pier 1 and 3 are aligned in one row. Whereas four bearings located atop Pier 2 are aligned in two rows. At Pier 1, the bearings are pinned. For Pier 2 and Pier 3, the bearings are constrained (between the pier and superstructure) for the global X direction and Z direction translational degrees of freedom. However, for translations in the global Y direction, each bearing of Pier 2 and Pier 3 is assigned a stiffness of 11 kips/in. All rotational degrees of freedom are released. Pile and pier structural responses in the bridge model are considered linear, and soil is not included in the model (for simplicity). The center node on pier cap at Pier 1 is subjected to, in the global Y direction, a prescribed displacement of 3 in.





File(s): *Example_3-2.in*

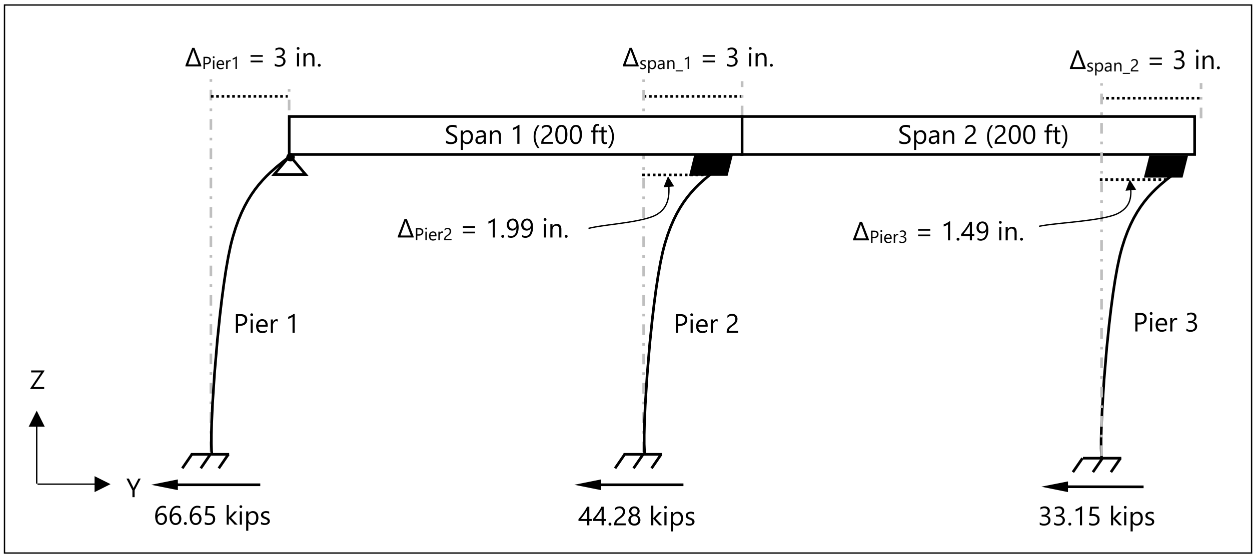


Figure 3.2 – Model Schematic for Example 3.2

A prescribed displacement (Δ_{Pier1} = 3 in.) is imposed in the global Y direction at the center of the Pier 1 pier cap. The prescribed displacement induces a pile base shear of R_{Pier1} = 66.65 kips (as obtained from FB-MultiPier). The stiffness of Pier 1 in the global Y direction is calculated as:

$K_{Pier1} = R_{Pier1} / \Delta_{Pier1}$ = 66.65 kips / 3 in. = 22.22 kips/in.

The structural configuration of Pier 2 is identical to that of Pier 1. Additionally, each of the four bearings atop Pier 2 is assigned a global Y direction bearing stiffness of 11 kips/in. The total stiffness of Pier 2 in the global Y direction is:

$1/ K_{Pier2} = 1/ K_{Pier1} + 1/ (4 \cdot 11$ kips/in.) = 1/ (22.22 kip/in.) + 1/ (44 kips/in.)
K_{Pier2} = 14.76 kips/in.

Similarly, the stiffness of Pier 3 in the global Y direction is:

K_{Pier3} = 11.05 kips/in.

Due to the prescribed displacement, bridge span 1 (Δ_{span1}) and span 2 (Δ_{span2}), displace 3 in (in the global Y direction):

$\Delta_{span1} = \Delta_{span2}$ = 3 in.

The corresponding pile base shear for Pier 2 (in the global Y direction) is:

R_{Pier2} = (Stiffness of Pier 2, K_{Pier2}) $\cdot$ (prescribed displacement at span end)
= (14.76 kips/in.) $\cdot$ (3 in.)
= 44.28 kips

Similarly, pile base shear for Pier 3 (in the global Y direction) is:

R_{Pier3} = 33.15 kips

Displacements that develop in the global Y direction at the pier caps of Pier 2 and Pier 3 (as obtained from FB-MultiPier) are:

Δ_{Pier2} =1.99 in. Δ_{Pier3} = 1.49 in.

Therefore, the global Y direction displacement ($\Delta_{bearing2}$) per bearing location at Pier 2 is:

$\Delta_{bearing2} = \Delta_{span2} - \Delta_{Pier2}$ = 3 in – 1.99 in. = 1.01 in.

Recall that the stiffness of each bearing pad, for translations in the global Y direction, is 11 kips/in. The corresponding bearing reaction (R_{B2}) acting on per bearing location at Pier 2 is:

R_{B2} = - ($\Delta_{bearing2}$) • (bearing stiffness)
= - (1.01 in.) • (11 kips/in.)
= -11.01 kips

Similarly, the global Y direction displacement ($\Delta_{bearing3}$) and bearing reaction (R_{B3}) acting per bearing location at Pier 3 are:

$\Delta_{bearing3}$ =1.51 in. R_{B3} = -16.61 kips

Results from FB-MultiPier Output File (Example_3-2.out):

```
BEARING REACTIONS - Pier #2
Bearing pad reactions are oriented local to the pad rotation

LOC       CASE          FX          FY          FZ         MXX         MYY         MZZ
                      kips        kips        kips      kip-ft      kip-ft      kip-ft
  1    L      1  -2.2923E-03 -1.1042E+01 -4.4940E-07  4.0713E-09  5.3191E-19 -1.9451E-11
  2    L      1   2.2923E-03 -1.1042E+01 -4.4940E-07  4.0713E-09 -4.4591E-18  1.9451E-11
  1    R      1   2.2923E-03 -1.1042E+01  4.5412E-07  4.0713E-09 -3.5488E-18 -1.9451E-11
  2    R      1  -2.2923E-03 -1.1042E+01  4.5412E-07  4.0713E-09  1.5124E-18  1.9451E-11
```

Reaction per bearing (FY) = -11.04 kips (matches R_{B2} within 1%)

```
BEARING REACTIONS - Pier #3
Bearing pad reactions are oriented local to the pad rotation

LOC       CASE          FX          FY          FZ         MXX         MYY         MZZ
                      kips        kips        kips      kip-ft      kip-ft      kip-ft
  1           1  2.1219E-10 -1.6522E+01 -4.1020E-09  2.9621E-09 -1.9407E-18 -1.4670E-11
  2           1  2.1274E-10 -1.6522E+01 -4.1020E-09  2.9621E-09 -1.9934E-18  1.4670E-11
```

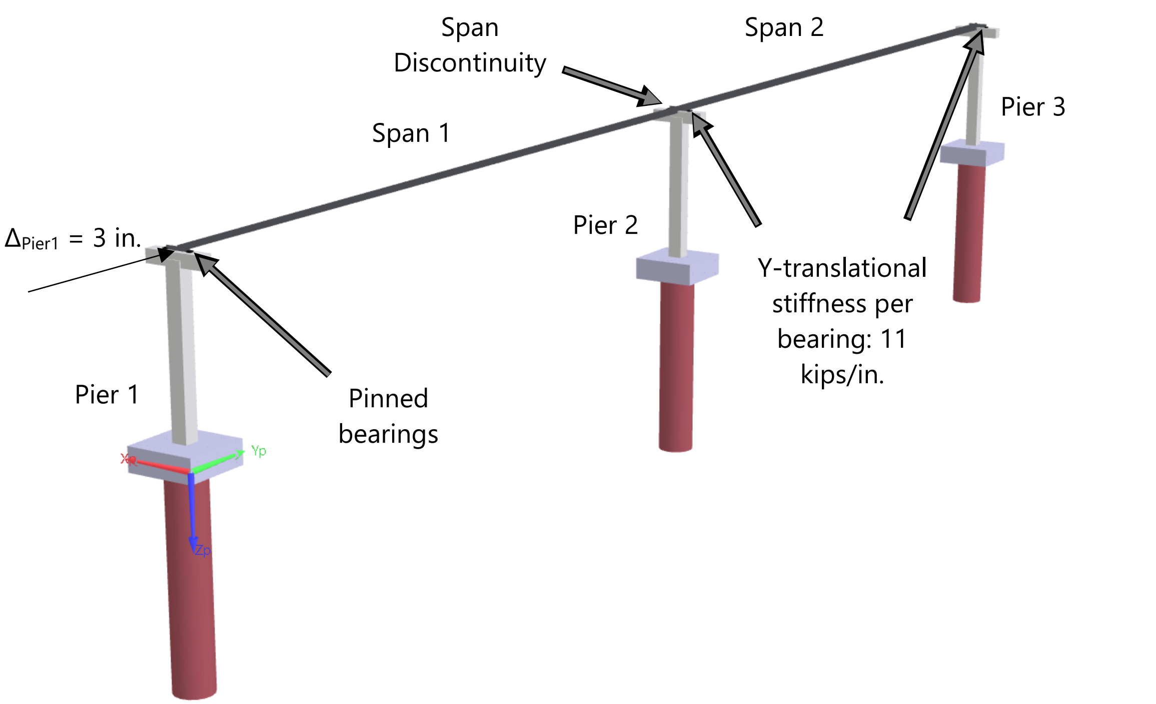
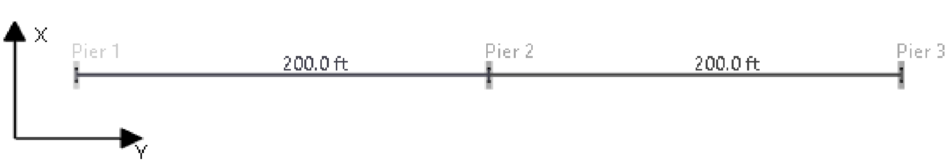
Reaction per bearing (FY) = -16.52 kips (matches R_{B3} within 1%)

```
Forces Acting on Tip Springs (Pier #2)

Forces are in units of  kips
Moments are in units of kip-ft

PILE NODE LOAD         Zp          Xp          Yp         RZp         RXp         RYp
   1  166    1     -0.000      -0.000      44.169       0.000    3975.210       0.000
```

Pile base shear at Pier 2 = 44.17 kips (matches R_{Pier2} within 0.5%)

```
Forces Acting on Tip Springs (Pier #3)

Forces are in units of  kips
Moments are in units of kip-ft

PILE NODE LOAD         Zp          Xp          Yp         RZp         RXp         RYp
   1  136    1     -0.000       0.000      33.044      -0.000    2973.980      -0.000
```

Pile base shear at Pier 3 = 33.04 kips (matches R_{Pier3} within 0.5%)

Example 3-3: Prescribed Displacements – Discontinuous Spans
Problem Description: The bridge piers shown below are not skewed and are simply supported, with discontinuous span lengths of 200 ft. Two bearings located atop Pier 1 and Pier 3 are aligned in one row. In contrast, four bearings are located atop Pier 2, and are aligned in two rows. At Pier 1, the bearings are pinned. For Pier 2 and Pier 3, the bearings are constrained (between the pier and superstructure) for the global X direction and Z direction translational degrees of freedom. However, for translations in the global Y direction, each bearing of Pier 2 and Pier 3 is assigned a stiffness of 11 kips/in. All rotational degrees of freedom are released. Pile and pier structural responses in the bridge model are considered linear, and soil is not included in the model (for simplicity). The center node on pier cap at Pier 1 is subjected to, in the global Y direction, a prescribed displacement of 3 in.


File(s): *Example_3-3.in*

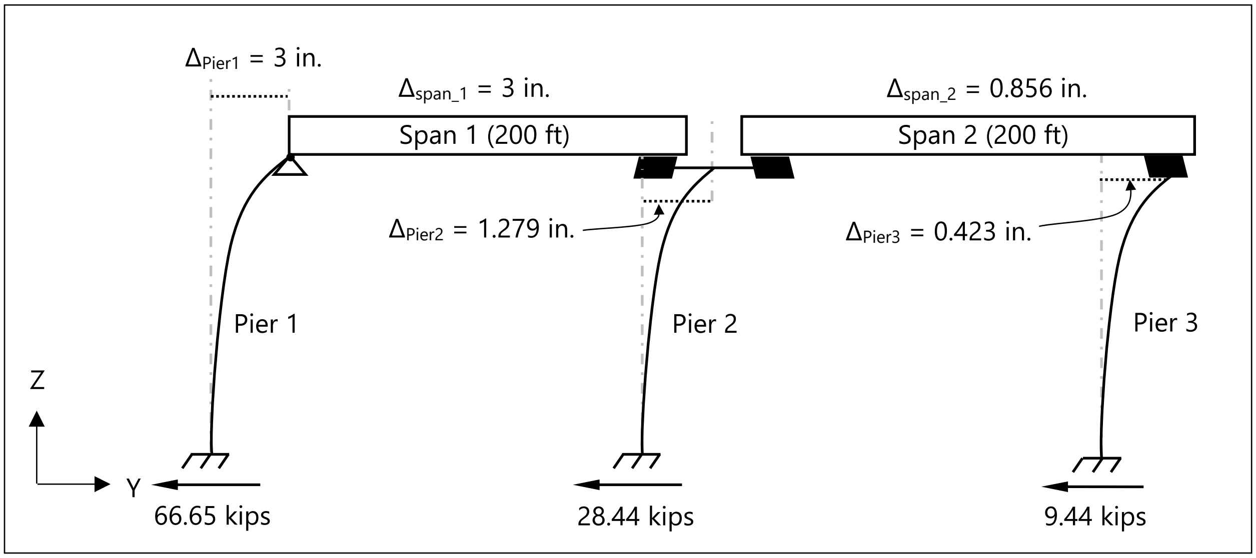


Figure 3.3 – Model Schematic for Example 3.3

A prescribed displacement (Δ_{Pier1} = 3 in.) is imposed in the global Y direction at the center of the Pier 1 pier cap. The prescribed displacement induces a pile base shear of R_{Pier1} = 66.65 kips (as obtained from FB-MultiPier). The stiffness of Pier 1 in the global Y direction is calculated as:

$K_{Pier1} = R_{Pier1} / \Delta_{Pier1}$ = 66.65 kips / 3 in. = 22.22 kips/in.

The structural configuration of Pier 3 is identical to that of Pier 1. Additionally, in the global Y direction, each of the two bearings atop Pier 3 is assigned a bearing stiffness of 11 kips/in. The global Y direction stiffness of Pier 3 is:

$1/ K_{Pier3} = 1/ K_{Pier1} + 1/ (2 \cdot 11$ kips/in.) = 1/ (22.22 kip/in.) + 1/ (22 kips/in.)
K_{Pier3} = 11.05 kips/in.

Similarly, the stiffness of Pier 2 in the global Y direction is:

K_{Pier2} = 14.76 kips/in.

Displacements that develop in the global Y direction at the pier caps of Pier 2 and Pier 3 (as obtained from FB-MultiPier) are:

Δ_{Pier2} =1.279 in. Δ_{Pier3} = 0.423 in.

Due to the prescribed displacement, bridge span 1 (Δ_{span1}) displace 3 in (in the global Y direction):

Δ_{span1} = 3 in.

The discontinuity in the bridge span causes the bridge span 2 to displace in the global Y direction:

$\Delta_{span2} = \Delta_{Pier2} - \Delta_{Pier3}$ = 1.279 in. - 0.423 in. = 0.856 in.

The global Y direction displacement ($\Delta_{bearing3}$) per bearing location at Pier 3 is:

$\Delta_{bearing3} = \Delta_{Pier3}$ = 0.423 in.

Recall that the stiffness of each bearing pad, for translations in the global Y direction, is 11 kips/in. The corresponding bearing reaction (R_{B3}) acting per bearing location at Pier 2 is:

R_{B3} = - ($\Delta_{bearing3}$) • (bearing stiffness)
= - (0.423 in.) • (11 kips/in.)
= - 4.65 kips

Similarly, the global Y direction displacement ($\Delta_{bearing2\_R}$) per right row bearing location at Pier 2 is:

$\Delta_{bearing2\_R} = \Delta_{span2} - \Delta_{Pier2}$ = 0.856 in. - 1.279 in. = -0.423 in.

The corresponding bearing reaction (R_{B2_R}) acting per right row bearing location at Pier 2 is:

R_{B2_R} = - ($\Delta_{bearing2\_R}$) • (bearing stiffness)
= - (-0.423 in.) • (11 kips/in.)
= 4.72 kips

The global Y direction displacement ($\Delta_{bearing2\_L}$) per left row bearing location at Pier 2 is:

$\Delta_{bearing2\_L} = \Delta_{span1} - \Delta_{Pier2}$ = 3 in. - 1.279 in. = 1.721 in.

The corresponding bearing reaction (R_{B2_R}) acting per right row bearing location at Pier 2 is:

R_{B2_L} = - ($\Delta_{bearing2\_L}$) • (bearing stiffness)
= - (1.721 in.) • (11 kips/in.)
= -18.93 kips

The global Y direction pile base shear for Pier 2 is equal to the resultant of the left and right row bearing reactions:

R_{Pier2} = - [(R_{B2_L}) • (No. of bearing) + (R_{B2_R}) • (No. of bearing)]
= - [(-18.93) • (2) + (4.72) • (2)]
= 28.42 kips

The global Y direction pile base shear for Pier 3 is:

R_{Pier3} = - (R_{B3}) • (No. of bearing)
= - (-4.72 kips) • (2)
= 9.44 kips

Results from FB-MultiPier Output File (Example_3-3.out):

```
 BEARING REACTIONS - Pier #2
 Bearing pad reactions are oriented local to the pad rotation

 LOC        CASE          FX          FY           FZ          MXX          MYY          MZZ
                        kips        kips         kips       kip-ft       kip-ft       kip-ft
   1    L      1  -1.4736E-03 -1.8897E+01 -4.5747E-11  2.5081E-09 -1.8474E-18 -1.2670E-11
   2    L      1   1.4736E-03 -1.8897E+01 -4.5745E-11  2.5081E-09 -1.8476E-18  1.2670E-11
   1    R      1   1.4736E-03  4.6997E+00  1.8376E-11  2.7468E-09  2.7613E-17 -1.2339E-11
   2    R      1  -1.4736E-03  4.6997E+00  1.8352E-11  2.7468E-09  2.7614E-17  1.2339E-11
```

Reaction per bearing in Left Row (FY) = -18.90 kips (matches R_{B2_L} within 0.2%)

Reaction per bearing in Right Row (FY) = 4.70 kips (matches R_{B2_R} within 0.5%)

```
 BEARING REACTIONS - Pier #3
 Bearing pad reactions are oriented local to the pad rotation

 LOC        CASE          FX          FY           FZ          MXX          MYY          MZZ
                        kips        kips         kips       kip-ft       kip-ft       kip-ft
   1           1  2.7413E-11 -4.6997E+00 -1.8184E-11  8.5276E-10  2.7254E-17 -4.1728E-12
   2           1  2.7568E-11 -4.6997E+00 -1.8207E-11  8.5276E-10  2.7254E-17  4.1728E-12
```

Reaction per bearing (FY) = -4.70 kips (matches R_{B3} within 0.5%)

```
Forces Acting on Tip Springs (Pier #2)

 Forces are in units of  kips
 Moments are in units of kip-ft

 PILE NODE LOAD         Zp         Xp         Yp         RZp         RXp         RYp
    1  166    1     -0.000     -0.000     28.394       0.000    2555.479       0.000
```

Pile base shear at Pier 2 = 28.40 kips (matches R_{Pier2} within 0.1%)

```
Forces Acting on Tip Springs (Pier #3)

 Forces are in units of  kips
 Moments are in units of kip-ft

 PILE NODE LOAD         Zp         Xp         Yp         RZp         RXp         RYp
    1  136    1     -0.000      0.000      9.399      -0.000     845.940      -0.000
```

Pile base shear at Pier 3 = 9.40 kips (matches R_{Pier3} within 0.5%)

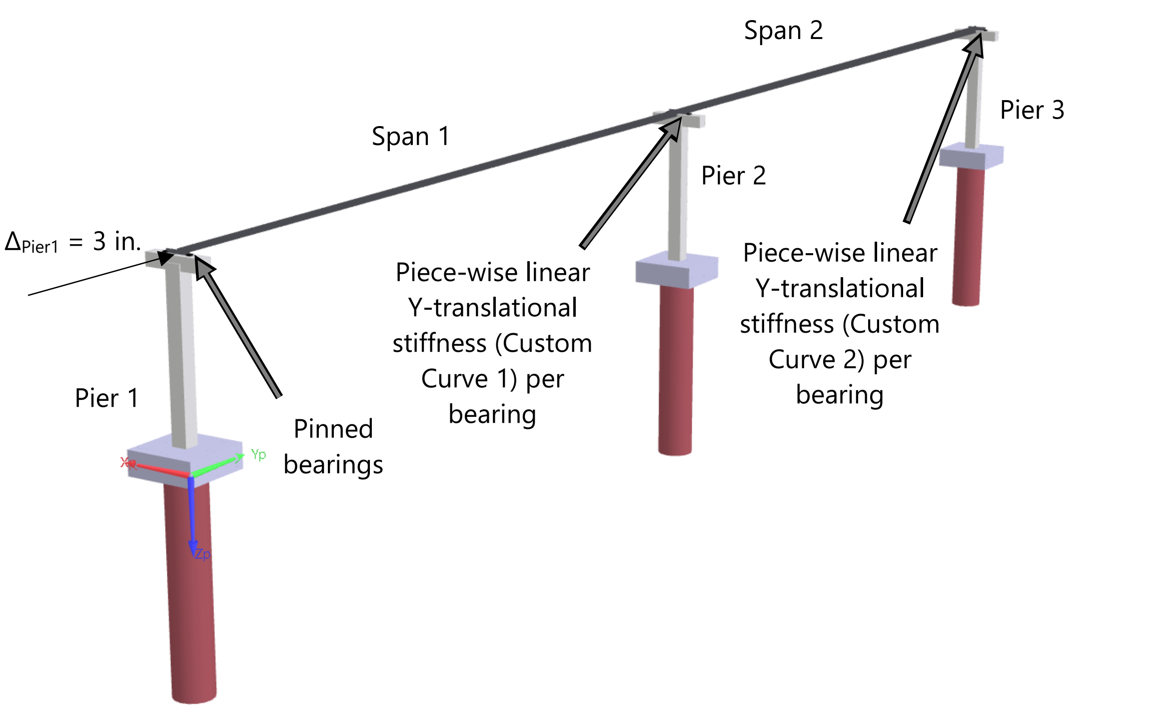
Chapter 4

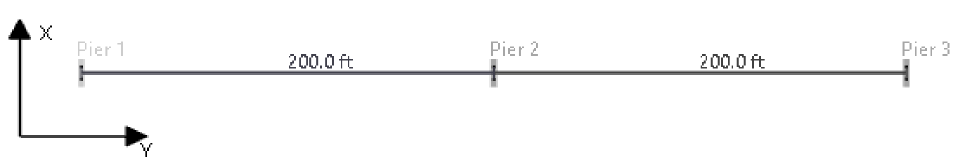
Two-Span Models with Non-Linear Bearings

In this chapter, three-substructure (two-span) models are developed with both pinned and custom (non-linear) bearings and are subjected to varying loading conditions, resulting in displacements along the global Y axis.

Example 4-1: Prescribed Displacements

Problem Description: The bridge piers shown below are not skewed and are simply supported, with two bearings per pier and span lengths of 200 ft. At Pier 1, the bearings are pinned. For Pier 2 and Pier 3, the bearings are constrained (between the pier and superstructure) for the global X direction and Z direction translational degrees of freedom. However, for translations in the global Y direction, each bearing of Pier 2 and Pier 3 is assigned a piece-wise linear stiffness. All rotational degrees of freedom are released. Pile and pier structural responses in the bridge model are considered linear, and soil is not included in the model (for simplicity). The center node on pier cap at Pier 1 is subjected to, in the global Y direction, a prescribed displacement of 5 in.





File(s): *Example_4-1.in*

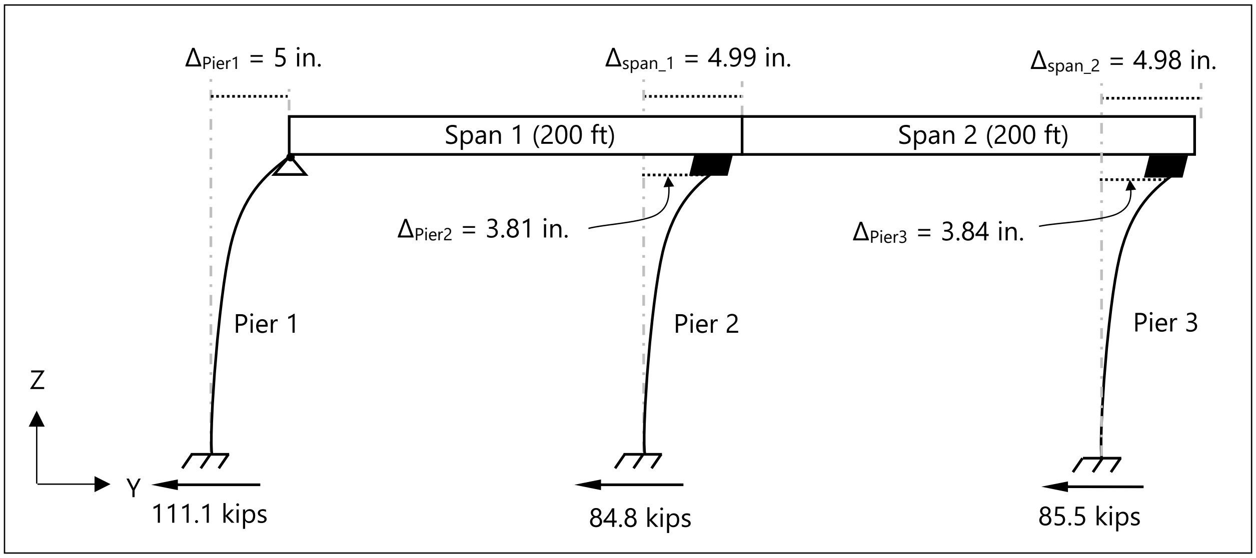


Figure 4.1 – Model Schematic for Example 4.1

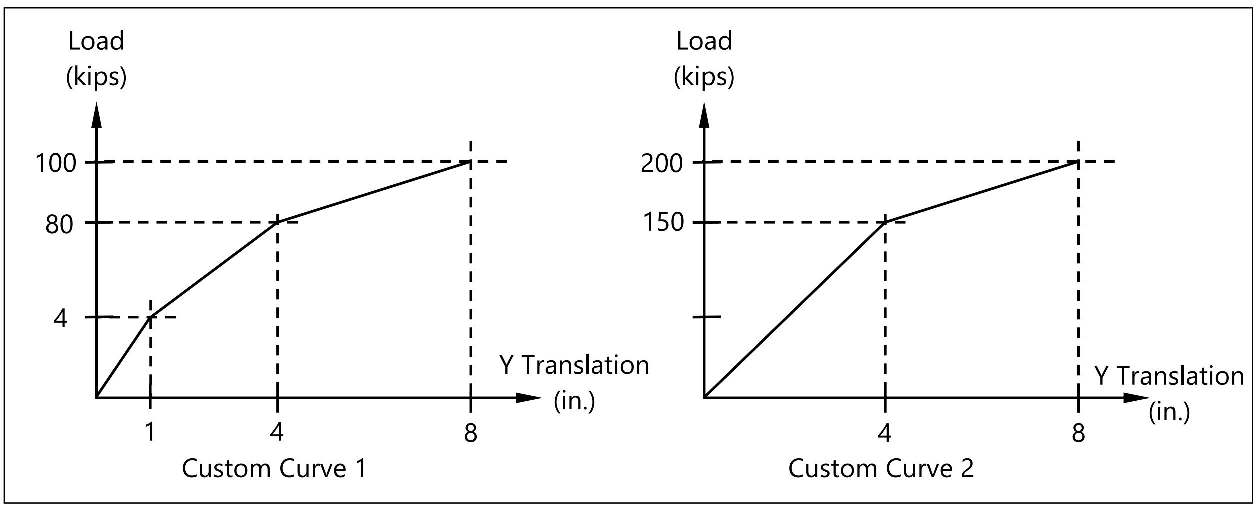


Figure 4.2 – Piece-wise Linear Y-Translational Stiffness Curve Assigned per Bearing atop Pier 2 (Left) and Pier 3 (Right)

A prescribed displacement (Δ_{Pier1} = 5 in.) is imposed in the global Y direction at the center of the Pier 1 pier cap. The prescribed displacement in Pier 1 induces a pile base shear of R_{Pier1} = 111.1 kips (as obtained from FB-MultiPier). The stiffness of Pier 1 in the global Y direction is calculated as:

$K_{Pier1} = R_{Pier1} / \Delta_{Pier1}$ = 111.1 kips / 5 in. = 22.22 kips/in.

Displacements that develop in the global Y direction at bridge span 1 and span 2 (as obtained from FB-MultiPier) are:

Δ_{span1} = 4.99 in. Δ_{span2} = 4.98 in.

The corresponding pile base shear for Pier 1 (in the global Y direction) is:

R_{Pier1} = (Stiffness of Pier 1, K_{Pier1}) • (prescribed displacement at span end)
= (22.22 kips/in.) • (5 in.)
= 111.1 kips

Displacements that develop in the global Y direction at the pier caps of Pier 2 and Pier 3 (as obtained from FB-MultiPier) are:

Δ_{Pier2} =3.81 in. Δ_{Pier3} = 3.84 in.

Therefore, the global Y direction displacement ($\Delta_{bearing2}$) per bearing location at Pier 2 is:

$\Delta_{bearing2} = \Delta_{span1} - \Delta_{Pier2}$ = 4.99 in. – 3.81 in. = 1.18 in.

Recall the piece-wise linear (Custom Curve 1) stiffness assigned to each bearing pad atop Pier 2, for translation in the global Y direction. The corresponding bearing reaction (R_{B2}) acting per bearing location is:

R_{B2} = - [(1 in.) • (40 kips/in.) + ($\Delta_{bearing2}$ – 1 in.) • (13.33 kips/in.)]
= - [(1 in.) • (40 kips/in.) + (1.18 in. – 1 in.) • (13.33 kips/in.)]
= -42.4 kips

The global Y direction displacement ($\Delta_{bearing3}$) per bearing location at Pier 3 is:

$\Delta_{bearing3} = \Delta_{span2} - \Delta_{Pier3}$ = 4.98 in. – 3.84 in. = 1.14 in.

Recall the piece-wise linear (Custom Curve 2) stiffness assigned to each bearing pad atop Pier 3, for translation in the global Y direction. The corresponding bearing reaction (R_{B3}) acting per bearing location:

R_{B3} = - ($\Delta_{bearing3}$) • (37.5 kips/in.)
= - (1.14 in.) • (37.5 kips/in.)
= -42.75 kips

The corresponding pile base shears for Pier 2 and Pier 3 (in the global Y directions) are:

R_{Pier2} = - (R_{B2}) • (No. of bearings)
= - (-42.4 kips) • (2)
= 84.8 kips

R_{Pier3} = - (R_{B3}) • (No. of bearings)
= - (-42.75 kips) • (2)
= 85.5 kips

Results from FB-MultiPier Output File (Example_4-1.out):

```
BEARING REACTIONS - Pier #2
Bearing pad reactions are oriented local to the pad rotation

LOC        CASE          FX          FY            FZ          MXX          MYY          MZZ
                       kips        kips          kips       kip-ft       kip-ft       kip-ft
  1           1  1.2058E-13 -4.2315E+01  7.9478E-03  1.5600E-08 -5.8103E-14 -7.4538E-11
  2           1  1.2058E-13 -4.2315E+01  7.9478E-03  1.5600E-08  5.8103E-14  7.4538E-11
```

Reaction per bearing (FY) = -42.32 kips (matches R_{B2} within 0.5%)

```
BEARING REACTIONS - Pier #3
Bearing pad reactions are oriented local to the pad rotation

LOC        CASE          FX          FY            FZ          MXX          MYY          MZZ
                       kips        kips          kips       kip-ft       kip-ft       kip-ft
  1           1  9.6114E-14 -4.2638E+01 -3.9739E-03  7.6450E-09  2.5579E-14 -3.7858E-11
  2           1  9.6112E-14 -4.2638E+01 -3.9739E-03  7.6450E-09 -2.5579E-14  3.7858E-11
```

Reaction per bearing (FY) = -42.64 kips (matches R_{B3} within 0.5%)

```
Forces Acting on Tip Springs (Pier #2)

Forces are in units of  kips
Moments are in units of kip-ft

PILE NODE LOAD         Zp          Xp          Yp         RZp         RXp         RYp
   1  136    1     -0.016       0.000      84.619      -0.000    7615.703      -0.000
```

Pile base shear at Pier 2 = 84.62 kips (matches R_{Pier2} within 0.5%)

```
Forces Acting on Tip Springs (Pier #3)

Forces are in units of  kips
Moments are in units of kip-ft

PILE NODE LOAD         Zp          Xp          Yp         RZp         RXp         RYp
   1  136    1      0.008       0.000      85.276      -0.000    7674.832       0.000
```

Pile base shear at Pier 3 = 85.28 kips (matches R_{Pier3} within 0.5%)

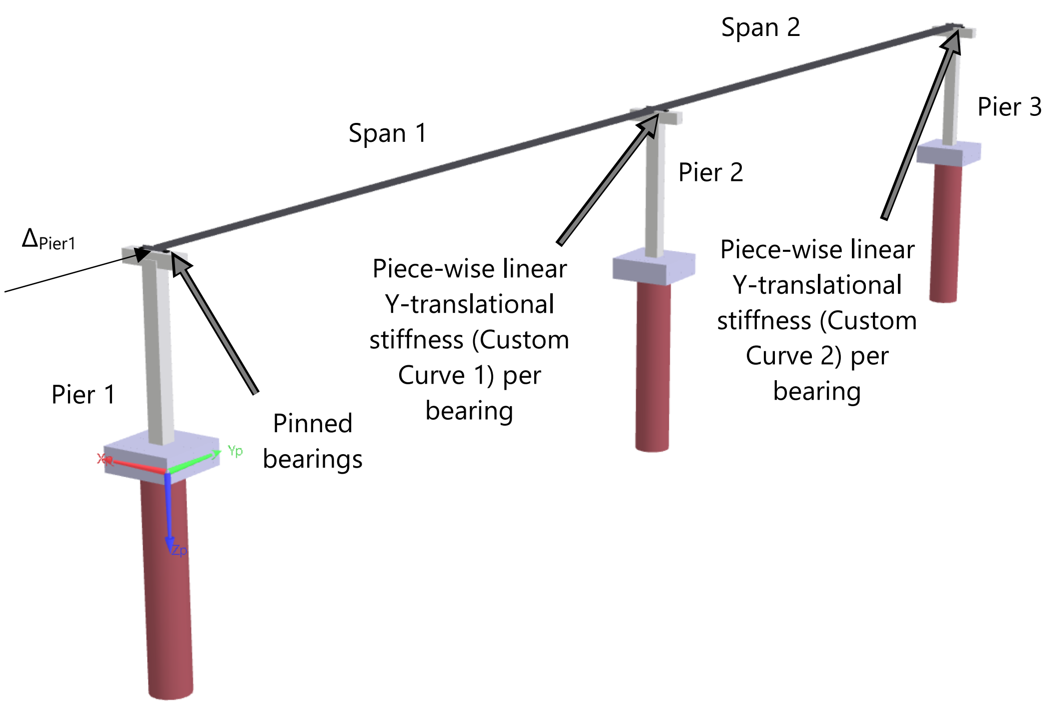
Chapter 5

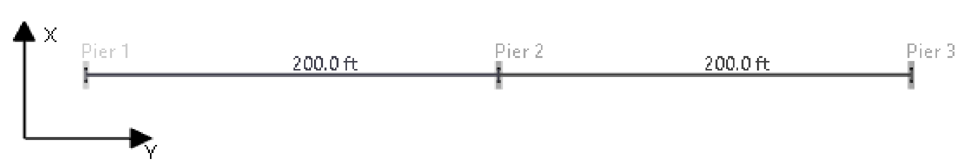
Two-Span Models with Displacement Control Bearings

In this chapter, three-substructure (two-span) models are developed with pinned bearings, custom (non-linear) bearings, and displacement control (non-linear) bearings. These models are subjected to varying loading conditions, resulting in displacements along the global Y axis.

Example 5-1: Prescribed Displacements

Problem Description: The bridge piers shown below are not skewed and are simply supported, with two bearings per pier and span lengths of 200 ft. At Pier 1, the bearings are pinned. At Pier 2 and Pier 3, the bearings are constrained between the pier and superstructure for the global X direction and Z direction translational degrees of freedom. However, for translations in the global Y direction, each bearing of Pier 2 and Pier 3 is assigned a piece-wise linear stiffness. Bearings atop Pier 2 mimic displacement control bearings. All rotational degrees of freedom are released for all bearings. Pile and pier structural responses in the bridge model are considered linear, and soil is not included in the model (for simplicity). The center pier cap node of Pier 1 is subjected to, in the global Y direction, prescribed displacements of 1.2 in, 2.4 in and 3.6 in (across three load cases).





File(s): *Example_5-1.in*

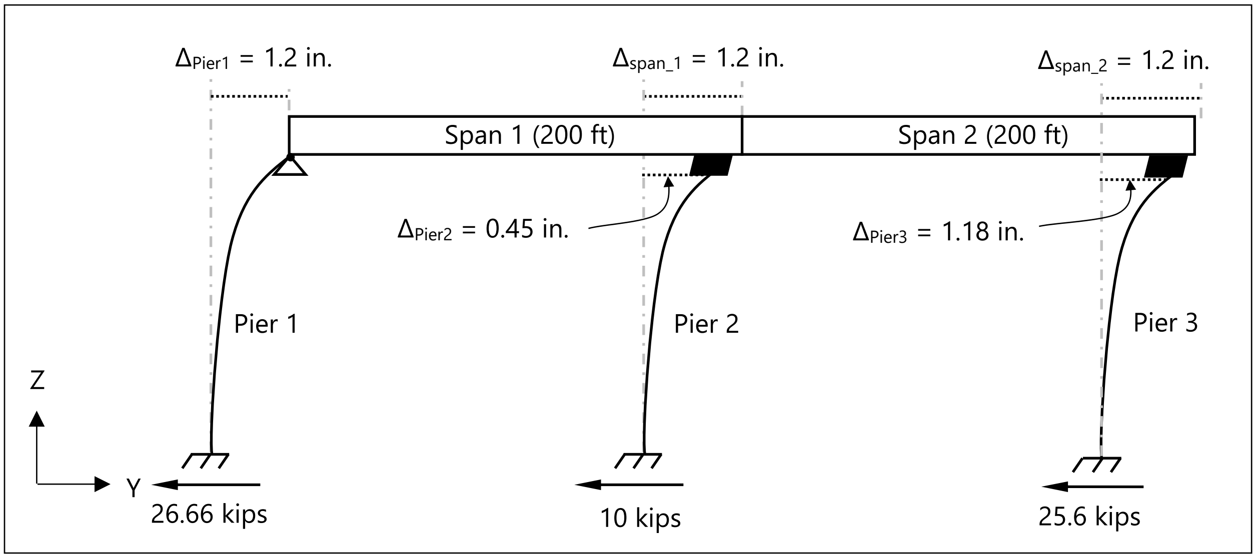


Figure 5.1 – Model Schematic for Example 5.1

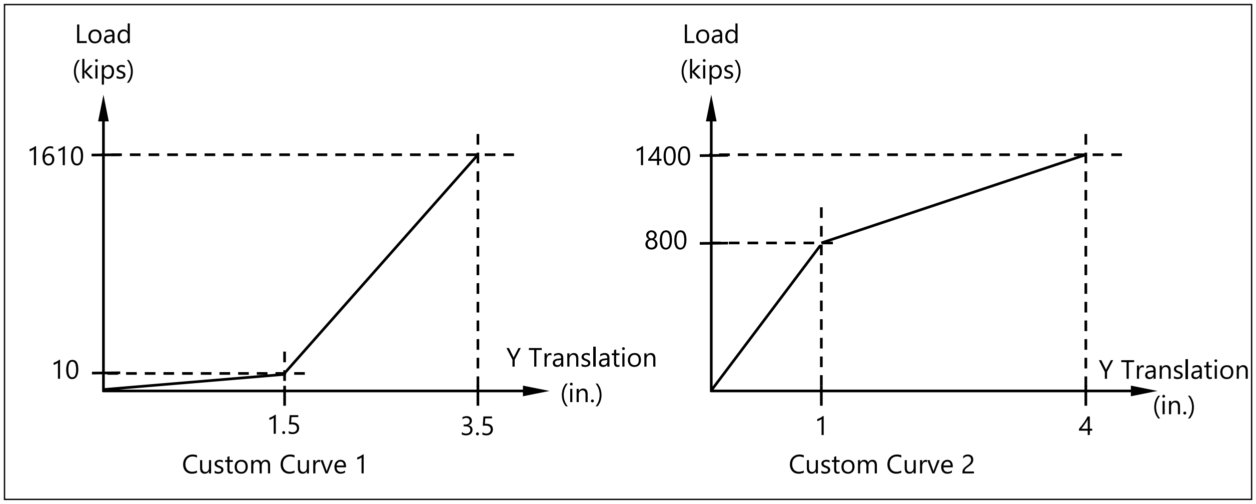


Figure 5.2 – Piece-wise Linear Y-Translational Stiffness Curve Assigned per Bearing atop Pier 2 (Left, Displacement Control) and Pier 3 (Right)

For load case 1, a prescribed displacement (Δ_{Pier1} = 1.2 in.) is imposed in the global Y direction at the center of the Pier 1 pier cap. The prescribed displacement in Pier 1 induces a pile base shear of R_{Pier1} = 26.66 kips (as obtained from FB-MultiPier). The stiffness of Pier 1 in the global Y direction is calculated as:

$K_{Pier1} = R_{Pier1} / \Delta_{Pier1}$ = 26.66 kips / 1.2 in. = 22.22 kips/in.

Displacements that develop in the global Y direction at bridge span 1 and span 2 (as obtained from FB-MultiPier) are:

Δ_{span1} = 1.2 in. Δ_{span2} = 1.194 in.

Displacements that develop in the global Y direction at the pier caps of Pier 2 and Pier 3 (as obtained from FB-MultiPier) are:

Δ_{Pier2} = 0.45 in. Δ_{Pier3} = 1.178 in.

Therefore, the global Y direction displacement ($\Delta_{bearing2}$) per bearing location at Pier 2 is:

$$\Delta_{bearing2} = \Delta_{span1} - \Delta_{Pier2} = 1.2 \text{ in.} – 0.45 \text{ in.} = 0.75 \text{ in.}$$

Recall the piece-wise linear (Custom Curve 1) stiffness assigned to each bearing pad atop Pier 2, for translation in the global Y direction. The corresponding bearing reaction (R_{B2}) acting per bearing location is:

$$R_{B2} = -(\Delta_{bearing2}) \cdot (6.67 \text{ kips/in.})$$
$$= -(0.75 \text{ in.}) \cdot (6.67 \text{ kips/in.})$$
$$= -5 \text{ kips}$$

The global Y direction displacement ($\Delta_{bearing3}$) per bearing location at Pier 3 is:

$$\Delta_{bearing3} = \Delta_{span2} - \Delta_{Pier3} = 1.194 \text{ in.} – 1.178 \text{ in.} = 0.016 \text{ in.}$$

Recall the piece-wise linear (Custom Curve 2) stiffness assigned to each bearing pad atop Pier 3, for translation in the global Y direction. The corresponding bearing reaction (R_{B3}) acting per bearing location:

$$R_{B3} = -(\Delta_{bearing3}) \cdot (800 \text{ kips/in.})$$
$$= -(0.016 \text{ in.}) \cdot (800 \text{ kips/in.})$$
$$= -12.8 \text{ kips}$$

The corresponding pile base shears for Pier 2 and Pier 3 (in the global Y directions) are:

$$R_{Pier2} = -(R_{B2}) \cdot (\text{No. of bearings})$$
$$= -(-5 \text{ kips}) \cdot (2)$$
$$= 10 \text{ kips}$$

$$R_{Pier3} = -(R_{B3}) \cdot (\text{No. of bearings})$$
$$= -(-12.8 \text{ kips}) \cdot (2)$$
$$= 25.6 \text{ kips}$$

Results from FB-MultiPier Output File (Example_5-1.out):

```
BEARING REACTIONS - Pier #2
Bearing pad reactions are oriented local to the pad rotation

LOC        CASE          FX         FY          FZ         MXX         MYY         MZZ
                       kips       kips        kips      kip-ft      kip-ft      kip-ft
  1           1  2.3924E-14 -4.9845E+00  1.9403E-03  1.8378E-09 -1.4185E-14 -8.7813E-12
  2           1  2.3924E-14 -4.9845E+00  1.9403E-03  1.8378E-09  1.4185E-14  8.7813E-12
```

Reaction per bearing (FY) = -4.98 kips (matches with R_{B2})

```
BEARING REACTIONS - Pier #3
Bearing pad reactions are oriented local to the pad rotation

LOC        CASE          FX         FY          FZ         MXX         MYY         MZZ
                       kips       kips        kips      kip-ft      kip-ft      kip-ft
  1           1  2.0306E-14 -1.3081E+01 -9.7014E-04  2.3455E-09  6.2445E-15 -1.1615E-11
  2           1  2.0305E-14 -1.3081E+01 -9.7014E-04  2.3455E-09 -6.2445E-15  1.1615E-11
```

Reaction per bearing (FY) = -13.08 kips (matches R_{B3} within 2.5%)

```
Forces Acting on Tip Springs (Pier #2)

Forces are in units of  kips
Moments are in units of kip-ft

PILE NODE LOAD         Zp         Xp         Yp         RZp        RXp        RYp
   1  136    1      -0.004      0.000      9.969      -0.000    897.209     -0.000
```

Pile base shear at Pier 2 = 9.97 kips (matches R_{Pier2} within 0.5%)

```
Forces Acting on Tip Springs (Pier #3)

Forces are in units of  kips
Moments are in units of kip-ft

PILE NODE LOAD         Zp         Xp         Yp         RZp        RXp        RYp
   1  136    1       0.002      0.000     26.163      -0.000   2354.665     -0.000
```

Pile base shear at Pier 3 = 26.16 kips (matches R_{Pier3} within 2.5%)

Similarly, we find the global Y direction displacement ($\Delta_{bearing2}$ and $\Delta_{bearing3}$) per bearing location at Pier 2 and Pier 3 for load case 2 and 3:

Load Case	**Prescribed displacement (global Y direction)**	**$\Delta_{bearing2}$ (Pier 2, displacement control bearing)**	**$\Delta_{bearing3}$ (Pier 3)**
1	1.2 in.	0.750 in.	0.016 in.
2	2.4 in.	1.503 in.	0.046 in.
3	3.6 in.	1.529 in.	0.070 in.

Commentary:

At Pier 2, the bearing initially translates 1.5 in. along the global Y direction with minimum resistance (6.67 kips/in.), but then is resisted by the increased stiffness of the bearing, (800 kips/in.). It can be seen that, through the use of displacement control bearings, one can mimic an expansion joint.

ACKNOWLEDGEMENT

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